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PAPERS

A fundamental approach: E-polarized electromagnetic wave diffraction by two dimensional arbitrary-shaped objects with impedance boundary condition

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In the present study, a new methodology in computational electromagnetics is developed for two-dimensional arbitrarily-shaped objects with impedance boundary conditions. The proposed approach investigates the E-polarized electromagnetic diffraction by a two-dimensional object with the Leontovich boundary condition. The scattered electric and magnetic fields are expressed as the convolution integral of the corresponding Green's function and the current induced on the obstacle surface. After obtaining integral equations by applying the boundary condition, the integral equations are solved as in the case of the method of auxiliary sources (MAS) which is a well-known method in computational electrodynamics. The results are compared with first, different methods such as the method of moments (MoM), orthogonal polynomials (OP), and second, different boundary conditions such as Dirichlet, Neumann, and fractional boundary conditions. Some results are also obtained for the different shape scatterers at some values of the surface impedance.

K e y w o r d s: computational electromagnetic, Green function, Dirichlet, Neumann, fractional boundary conditions

1 Introduction

Electromagnetic scattering by the scatterers with the impedance boundary conditions has crucial importance compared to the Dirichlet and Neumann boundary conditions since the limit cases of the impedance boundary condition may cover the aforementioned boundary conditions and between. In other words, the impedance boundary condition describes the real, conducting, lossy objects. In the literature known to us, it has been observed that the electromagnetic scattering problems with the impedance boundary condition in generals deal with particular geometries. The main aim of this study is to generalize the solution for any kind of geometries in 2D.

In the present study, a general approach for E-polarized electromagnetic diffraction by two-dimensional arbitrary-shaped objects with impedance boundary conditions using dyadic Green's function and Leontovich boundary condition is developed. The details of the mathematical formulations are provided in the appendix. The main advantages of the method are first to employ an integral equation approach by deploying the boundary conditions and second to obtain a compact expression for field components by using dyadic Green's function and most importantly to be valid for arbitrary-shaped objects. Later, the integral equation is solved as it has been done in MAS by shifting the current densities from the actual physical surface to an auxiliary surface to avoid the singularity problem [1], [2]. However, it should be highlighted that the boundary condition is satisfied on the actual surface.

To investigate various theoretical or practical surfaces, different boundary conditions have been proposed in the literature such as perfect electromagnetic conductor, impedance, sesquilinear, Dirichlet, Neumann, and fractional boundary conditions [3-6]. By defining the appropriate and legitimate boundary conditions for electrodynamic phenomena, the class of problems is enlarged not only in scattering theory but also, antenna, and microwave component designs. In the literature, there are numerous studies regarding electromagnetic scattering including impedance boundary conditions [6-10]. In these problems, flat surfaces with impedance or resistance values were investigated with different methodologies regarding the strip, slotted cylinder, and disk. In general, high-frequency asymptotics was employed to solve such kind of problems. Besides, circular geometries with impedance boundary conditions were also investigated by using the periodicity in angular directions [11-14]. The advantage of the proposed method is that the approach allows for the investigation of the diffraction by arbitrary-shaped objects with impedance boundary conditions.

Two-dimensional electromagnetic scattering problems with different boundary conditions play important role in various areas of electromagnetic theory and applications. First, new analytical, numerical, and semianalytical- numerical methods, in general, are considered in the solution of two-dimensional electromagnetic problems [15-17].

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There are several reasons that researchers focus on the topic such as developing faster converging numerical approaches [18-20], employing different boundary conditions [12], [21], and proposing new mathematical perspectives [22-25]. In [18], near-zero-index objects involved in electromagnetic problems are investigated through a novel surface integral equation formulation. Also, smoothing canonically parametrized contours for super-algebraically convergent algorithms is analyzed in [19]. Besides, the two-dimensional geometries intensively are studied not only for clarifying how the proposed methodologies' convergence is faster but also they are investigated how the electromagnetic wave behave in the vicinity of different boundary conditions or medium and resonances, reflections and radar cross section in such scenarios are examined [5], [12], [21]. In addition to these aforementioned reasons, two dimensional electromagnetic problems yield to develop or propose a novel or hybrid approaches in time and frequency domains [23], [24].

The presented paper is an example of such an attempt. The proposed approach considers arbitrary shape geometries, has fast convergence, and generalizes the boundary condition for the Method of Auxiliary sources to the impedance boundary condition for the first time. Besides, the proposed methodology is verified by analytical, numerical, and analytical-numerical methods in the present study.

The following sections are devoted to the mathematical formulation of the problem, numerical results, and comparison, then the conclusion is drawn.

2 Formulation of the problem

In this section, the mathematical background of the proposed method is provided. The scattered electric field vector ($\boldsymbol{E}_{\rm sc}$) and magnetic field vector ($\boldsymbol{H}_{\rm sc}$) are provided as, [26], [27]

$$\begin{aligned}\boldsymbol{E}_{\rm sc}(\boldsymbol{r}) &= \oiint_{S'} i\omega\mu\, \boldsymbol{J}_e(\boldsymbol{r}')\cdot\overline{\overline{G}}(\boldsymbol{r},\boldsymbol{r}')+ \\ &\quad +\boldsymbol{J}_m(\boldsymbol{r}')\cdot\nabla\times\overline{\overline{G}}(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}s' \\ \boldsymbol{H}_{\rm sc}(\boldsymbol{r}) &= -\oiint_{S'} i\omega\varepsilon\, \boldsymbol{J}_m(\boldsymbol{r})\cdot\overline{\overline{G}}(\boldsymbol{r},\boldsymbol{r}')- \\ &\quad \boldsymbol{J}_e(\boldsymbol{r}')\cdot\nabla\times\overline{\overline{G}}(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}s',\end{aligned} \tag{1}$$

where, $\boldsymbol{E}$ and $\boldsymbol{H}$ stand for the total electric and magnetic field vectors, respectively and

$$\begin{aligned}\overline{\overline{G}}(\boldsymbol{r},\boldsymbol{r}') &= \Big(\overline{\overline{I}}+\frac{\nabla\nabla}{k^2}\Big)G(\boldsymbol{r},\boldsymbol{r}')\,,\\ \boldsymbol{J}_m(\boldsymbol{r}') = \hat{\boldsymbol{n}}\times\boldsymbol{E}(\boldsymbol{r}'), &\quad \boldsymbol{J}_e(\boldsymbol{r}') = \hat{\boldsymbol{n}}\times\boldsymbol{H}(\boldsymbol{r}').\end{aligned}$$

Here, $\overline{\overline{G}}$ and G are the dyadic and scalar Green's function, respectively, $\boldsymbol{J}_{\rm m}(r')$ and $\boldsymbol{J}_{\rm e}(r')$ are magnetic and electric current densities on the scatterer surface. Besides, $(\boldsymbol{r},\boldsymbol{r}')$ correspond to the observation and the source points, respectively in general, also $\hat{\boldsymbol{n}}$ stands for the normal unit vector of the scatterer surface, ω is the angular frequency, time dependency is given as $e^{-i\omega t}$ and finally, ε and μ are permittivity and permeability of the corresponding region. The general form of the Leontovich boundary condition is provided [12], [28]:

$$\boldsymbol{E}_t = \zeta_S \boldsymbol{H}_t\times\hat{\boldsymbol{n}}\,, \tag{2}$$

where $\boldsymbol{E}_t$ and $\boldsymbol{H}_t$ stand for the tangential components of the electric and magnetic fields, and $\zeta_s = R+iX$ is the effective surface impedance with resistivity (R) and the the effective (X) reactance, respectively.

For two-dimensional E-polarized cases $\boldsymbol{J}_{\rm e}$ has only z-component and $\boldsymbol{J}_{\rm m}$ becomes zero. We express the tangential unit vectors of the scatterer as $\hat{\boldsymbol{t}}_\perp$ and $\hat{\boldsymbol{t}}_\parallel$. Then, for this specific case, the scattered fields can be expressed as

$$\begin{aligned}\boldsymbol{E}_{\rm sc}(\boldsymbol{r}) &= \oiint_{S'} i\omega\mu\, J_e(\boldsymbol{r}')\overline{\overline{G}}(\boldsymbol{r},\boldsymbol{r}')\cdot\hat{\boldsymbol{t}}_\perp(\boldsymbol{r}')\mathrm{d}s',\\ \boldsymbol{H}_{\rm sc}(\boldsymbol{r}) &= \oiint_{S'} J_e(\boldsymbol{r}')\nabla\times\overline{\overline{G}}(r,\boldsymbol{r}')\cdot\hat{t}_\perp(\boldsymbol{r}')\mathrm{d}s',\end{aligned} \tag{3}$$

where $\boldsymbol{J}_e(\boldsymbol{r}') = J_e(\boldsymbol{r}')\hat{t}_\perp(\boldsymbol{r}')$.

Here, $G(\boldsymbol{r},\boldsymbol{r}') = -\frac{i}{4}H_0^{(1)}(\boldsymbol{r},\boldsymbol{r}')$, $H_0^{(1)}(\boldsymbol{r},\boldsymbol{r}')$ is the Hankel Function of the first kind and zero-th order.

Using (2), the tangential components of E and H-fields can be found

$$\begin{aligned}&E_\perp(\boldsymbol{r}) = \\ &\oiint_{S'} i\omega\mu J_e(\boldsymbol{r}')\overline{\overline{G}}(r,\boldsymbol{r}')\cdot\hat{t}_\perp(\boldsymbol{r}')\mathrm{d}s' + \boldsymbol{E}_{\rm inc}(\boldsymbol{r})\cdot\hat{t}_\perp(\boldsymbol{r}),\\ &\boldsymbol{H}_\parallel(\boldsymbol{r}) = \\ &\oiint_{S'} J_e(\boldsymbol{r}')\nabla\times\overline{\overline{G}}(\boldsymbol{r},\boldsymbol{r}')\cdot\hat{t}_\perp(\boldsymbol{r}')\mathrm{d}s' + \boldsymbol{H}_{\rm inc}(\boldsymbol{r})\cdot\hat{t}_\parallel(\boldsymbol{r}).\end{aligned} \tag{4}$$

Here, $\boldsymbol{E}_{\rm inc}$, $\boldsymbol{H}_{\rm inc}$ are incident field components. It should be highlighted that $\boldsymbol{E}(\boldsymbol{r})\cdot\hat{t}_\parallel = 0$ and $\boldsymbol{H}(\boldsymbol{r})\cdot\hat{y}_\perp = 0$ since we investigate E-polarized diffraction by a two-dimensional object. Then, the boundary condition given in (2) is applied and the following equations are obtained

$$E_\perp(\boldsymbol{r}) = \zeta_S \boldsymbol{H}_\parallel(\boldsymbol{r}). \tag{5}$$

This integral equation is converted into a system of linear algebraic equations (SLAE) as below (replacing the integrals by summation)

$$\sum_{j=1}^{n}\big(A_{ij}-\zeta_s B_{ij}\big)J_e(\boldsymbol{r}'_j) = C_i, \tag{6}$$

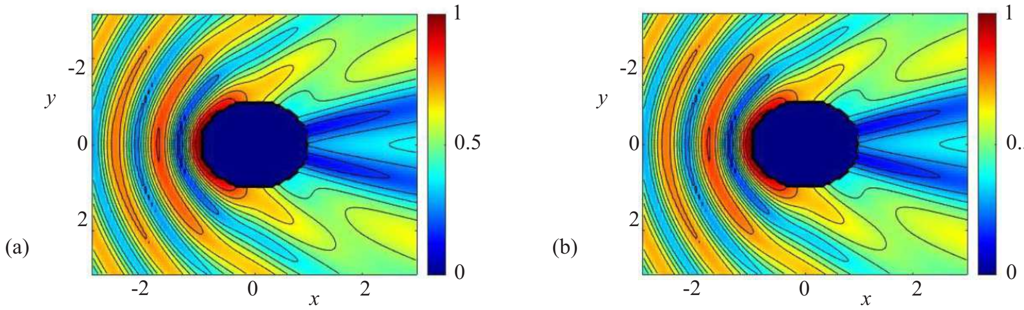

Fig. 1. The normalized electric field distributions for perfect magnetic conductor circular cylinder $\theta_0 = \pi$, $k = 4$, $a = 1$: (a) – analytical result, and (b): $\zeta_s = 1000 \times 120\pi$

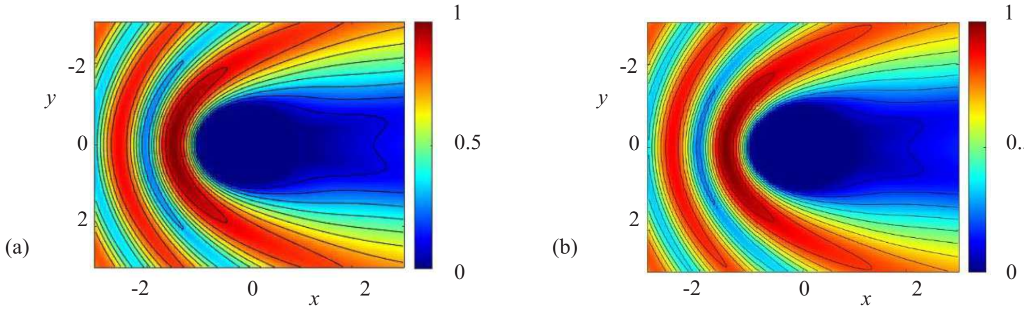

Fig. 2. The normalized electric field distributions for perfect electric conductor circular cylinder $\theta_0 = \pi$, $k = 3$, $a = 1$: (a) – (MoM), and (b): $\zeta_s = 0$

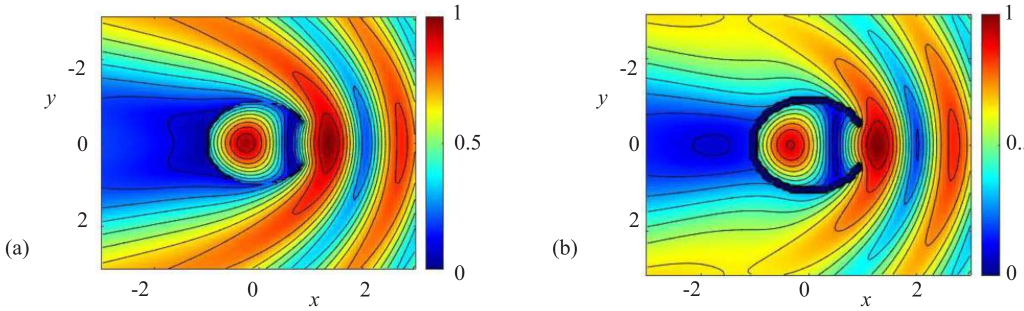

Fig. 3. The normalized electric field distributions for a circular arc with fractional boundary condition, $\theta_0 = 0$, $k = 2.3$, $a = 1$, and aperture size $= \pi/3$; (a) – fractional order is 0.5 and (b): $\zeta_s = -i120\pi$

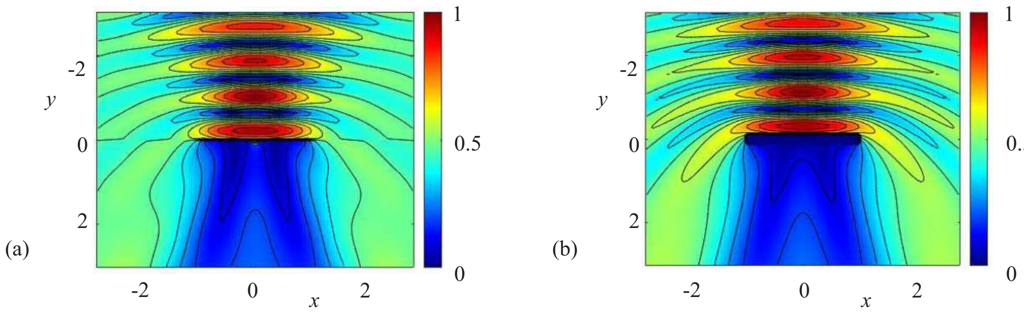

Fig. 4. The normalized electric field distributions for a circular arc with fractional boundary condition, $\theta_0 = \pi/2$, $k = 3.9$, $a = 1$, $\zeta_s = -i120\pi$ and fractional order is 0.5

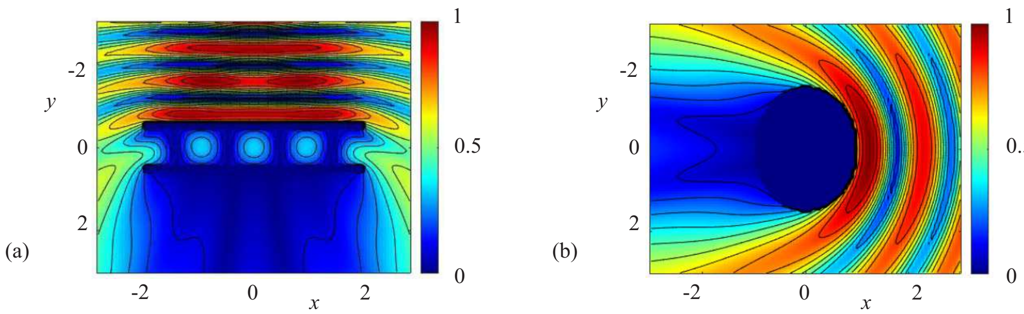

Fig. 5. The normalized electric field distributions for double strips with $\zeta_s = -i120\pi$ and (a): $\theta_0 = \pi/2$, $k = 4$, $a = 2$, $b = 0.5$, and elliptical cylinders (b): $\theta_0 = 0$, $k = 3$, $a = 1$, $b = 1.5$

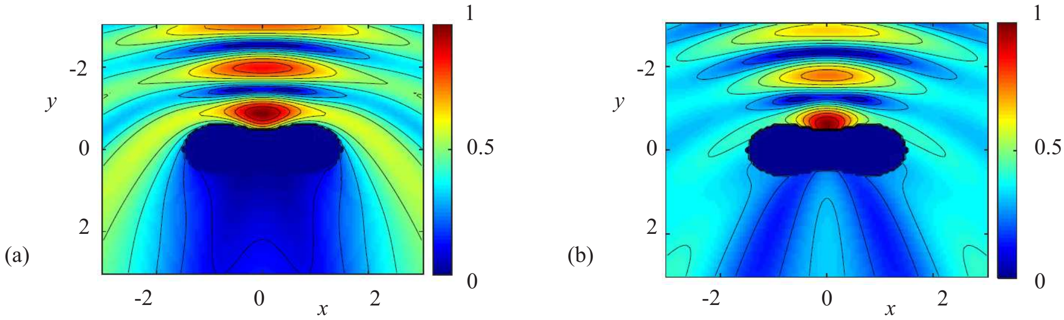

Fig. 6. The normalized electric field distributions for Cassini Oval Shaped Cylinders, [34],with $\theta_0 = \pi/2$, $k = 3$, $a = 1$, $b = 1.1$, and (a): $\zeta_s = -i120\pi$, (b): $\zeta_s = (0.4 - i10)120\pi$

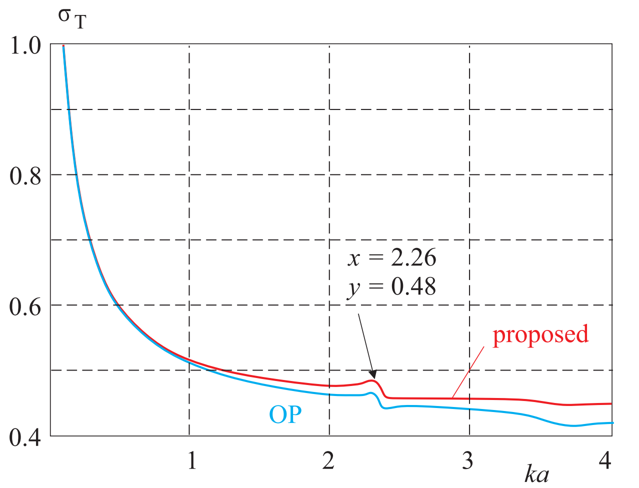

Fig. 7. Normalized total radar cross section for OP and proposed approach; $\theta_0 = 0$, perfect electric conducting surface, $a = 1$, aperture size = $\theta/3$

where

$$A_{ij} = i\omega\mu\overline{\overline{G}}(\mathbf{r}_i, \mathbf{r}'_j) \cdot \hat{t}_\perp(\mathbf{r}'_j) \cdot \hat{t}_\perp(\mathbf{r}_i),$$
$$B_{ij} = \nabla \times \overline{\overline{G}}(\mathbf{r}_i, \mathbf{r}'_j) \cdot \hat{t}_\perp(\mathbf{r}'_j) \cdot \hat{t}_\parallel(\mathbf{r}_i),$$
$$C_i = \zeta_s \boldsymbol{H}_{\text{inc}}(\mathbf{r}_i) \cdot \hat{t}_\parallel(\mathbf{r}_i) - \boldsymbol{E}_{\text{inc}}(\mathbf{r}_i) \cdot \hat{t}_\perp(\mathbf{r}_i).$$

It should be highlighted that the integral equation is solved by MAS where the induced current density is shifted from the actual surface to the auxiliary surface. Then, fast convergence is guaranteed and the singularity problems are eliminated [29]. Besides, it should be noticed that the integrals in the proposed approach stand for the closed surfaces. If it is required to investigate non-closed circular or elliptical arcs, the surface should have a tiny thickness compared to the electrical length and the normal vectors of its surfaces should have opposite directions on different sides. The analysis and comparison with other approaches for this case are also provided in the following section.

3 Numerical results and comparison with other methods

In this part, the validation of the proposed approach is provided by the numerical results for different geometries and also, and comparisons with analytical outcomes, MoM, and OP are done. As the incident field, an E-polarized plane wave is considered for all numerical results. It should be noted that the angle of the incidence is given as θ_0 from the x-axis, and the radius of the circle is given as a. Since the impedance boundary condition models the scatterer as a thin surface, the electric field inside the object is nullified. Only the field outside the object should be considered.

In Fig. 1, the analytical result [30] is compared for impedance boundary conditions where the effective impedance value takes huge values. As expected, the field outside the object matches with a very high degree of accuracy. This can be thought that one limit case of the Leontovich Boundary condition approaches the Neumann boundary condition.

In Fig. 2, the comparison is done with MoM for the case of perfect electric conductor surface (Dirichlet condition for E-polarized case). The deviation from MoM is less than 3%, [31].

In Fig. 3, the comparison is done for a non-closed circular arc. In Figure 3(a), a fractional boundary condition is employed [32]. It should be noted that, in the proposed method, the thickness of the circular arc is taken as 0.02 (thickness$\ll$k). It can be easily noticed that the resonance is observed inside the circular arc.

In Fig. 4, the comparison between the fractional boundary condition and impedance boundary condition is provided [33]. Here, the diffraction by the strip with 2 is investigated. The relation between the fractional order and the impedance value is derived for electrically large scatterers; therefore, the deviation in the vicinity of the strip plane is observed, otherwise, good agreement is observed as expected.

In Fig. 5, electromagnetic scattering from the different geometries is provided. In Fig. 5(a), the double strip is investigated with the length of $2a$ and the distance from each other is $2l$ whereas, in Fig. 5(b), electromagnetic scattering by an elliptical cylinder with radii, a and b, is provided.

In Fig. 6, the scattering by Cassini Oval cylinders is given for different impedance values. The impedance value on the surface changes the field distribution, drastically.

In Fig. 7, the total radar cross-section (σ_T) analysis of circular non-closing arc is investigated (we should specify the impedance value). In the figure, the comparison is done with OP, see Fig. 3 for the geometry, [12]. The resonance wavenumbers for closed circular cylinders are at Bessel's function zeros for the Dirichlet boundary condition. Since the circular arc has an aperture, the deviation from the zero of the Bessel function is expected. The deviation between the two methodologies is because the proposed approach solves the geometries with finite thickness whereas OP deals with obstacles with infinitesimal thickness. Therefore, when the frequency increases, the deviation is noticeable.

4 Conclusions

In the present study, two-dimensional electromagnetic diffraction by impedance surfaces with finite thickness for an E-polarized case is investigated by proposing a new mathematical approach. The advantages of the proposed approach are that the method can be employed for any two-dimensional geometries with impedance boundary conditions and the mathematical derivation and formulation are compact and neat since the dyadic Green's function is employed. The comparisons regarding the methodologies are done with analytical results, MoM, OP. Besides, limit cases of the Leontovich boundary condition and the comparison with fractional boundary condition yield that, the proposed approach is effective and valid for two-dimensional E-polarized scattering problems. The results reveal that, for all cases, the deviation from the other methods is less than 5%.

Appendix

The details on the mathematical manipulations are provided here for readers. The distance between the observation and the source point is provided below

$$\rho = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$
$$\Delta x = x' - x, \Delta y = y' - y, \Delta z = z' - z$$

Since we are involved in a two-dimensional problem, Green's function is the Hankel function of the first kind due to $e^{-i\omega t}$. Then, nine components of $\overline{\overline{G}}(\mathbf{r}, \mathbf{r}')$ in (6) are – for brevity, the argument of Hankel functions ($k\rho$) was omitted

$$G_{xx} = \frac{i}{4}\Big(- H_0^{(1)} + \frac{1}{k\rho}H_1^{(1)} - \frac{\Delta x^2}{\rho^2}H_2^{(1)}\Big),$$
$$G_{yy} = \frac{i}{4}\Big(- H_0^{(1)} + \frac{1}{k\rho}H_1^{(1)} - \frac{\Delta y^2}{\rho^2}H_2^{(1)}\Big),$$
$$G_{zz} = -\frac{i}{4}H_0^{(1)}, \quad G_{xy} = G_{yx} = \frac{i}{4}\frac{\Delta x \Delta y}{\rho^2}H_2^{(1)}$$
$$G_{xz} = G_{zx} = G_{yz} = G_{zy} = 0$$

Since we have a two-dimensional problem, the change in the z-direction is zero.

Then the nine components of tensor $\nabla \times \overline{\overline{G}}$ are given by the following matrix

$$\begin{bmatrix} 0 & 0 & \dfrac{\partial G_{xz}}{\partial y} \\ 0 & 0 & -\dfrac{\partial G_{zz}}{\partial x} \\ \dfrac{\partial G_{yx}}{\partial x} - \dfrac{\partial G_{xx}}{\partial y} & \dfrac{\partial G_{yy}}{x} - \dfrac{\partial G_{xy}}{y} & 0 \end{bmatrix}$$

where

$$\frac{\partial G_{xy}}{\partial y} - \frac{i}{4}\Delta x\Big[H_2^{(1)}\frac{\Delta y^2 - \Delta x^2}{\rho^4} - k\frac{\Delta y^2}{\rho^3}\Big\{\frac{2H_2^{(1)}}{k\rho} - H_3^{(1)}\Big\}\Big],$$
$$\frac{\partial G_{yx}}{\partial x} = -\frac{i}{4}\Delta y\Big[H_2^{(1)}\frac{\Delta x^2 - \Delta y^2}{(\rho)^4} - k\frac{\Delta x^2}{(\rho)^3}\Big\{\frac{2H_2^{(1)}}{k\rho} - H_3^{(1)}\Big\}\Big],$$
$$\frac{\partial G_{zz}}{\partial x} = \frac{\partial G_{yy}}{\partial x} = -\frac{ik}{4}\frac{\Delta x}{\rho}H_1^{(1)},$$
$$\frac{\partial G_{xx}}{\partial y} = \frac{\partial G_{zz}}{\partial y} = -\frac{ik}{4}\frac{\Delta y}{\rho}H_1^{(1)},$$

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