

Run length limited CCSDS convolutional codes for optical communications

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This paper presents the construction of RLL-ECCs (run length limited error control codes) from three selected ECCs specified by Consultative Committee for Space Data Systems (CCSDS) for optical communications. The RLL-ECCs obtained present a practical alternative to CCSDS codes with pseudo-randomizers. Their advantage is that the maximal run lengths of equal symbols in their codeword sequences are guaranteed, which is not the case if the common approach with pseudo-randomizers is used. The other advantages are that no additional redundancy is introduced into encoded codewords and that the encoding and decoding procedures of the original error control CCSDS codes do not have to be modified in the following cases: Firstly, if hard decoding is used and the transmission channel can be modeled as a BSC (binary symmetric channel) and secondly, if soft decoding and coherent BPSK (binary phase shift keying) modulation is used and the appropriate transmission channel model is an AWGN (additive white Gaussian noise) channel.

Key words: run length limited error control codes, randomizer, modifier, soft decoding, coherent BPSK

1 Introduction

The construction of codes adapted to different practical requirements can be traced back to the times when different line codes were proposed empirically based on experience. The Morse alphabet is probably the first code which was constructed taking into account the practical requirements or constraints of telegraphy channels at that time [1]. Later, after Shannon established Information Theory [2], more advanced codes for constrained channels and memory systems were developed step by step [3-9].

Simply expressed, in the digital domain, the purpose of these codes is to convert any sequence of symbols from some source into a sequence which satisfies the constraints of practical transmission channels or storage systems. These constraints stem from different practical requirements and goals. For example, the synchronization of the receiver could be supported by sufficient changes in transmitted signals. This could be satisfied by RLL codes, which form a subset of constrained codes. RLL codes are distinguished by the property that the runs of identical consecutive symbols are limited.

In practical up-to-date communications and storage systems the constrained codes are mostly used together with ECCs. The purpose of ECCs is to decrease the error probability in the payload information after its decoding. However, there is a fundamental problem of how to order the constrained codes and ECCs in cascade. The question is which code should be the inner code and which

the outer. It stems from the fact that constrained codes are not suitable for error prone channels. In theory it is supposed that the constrained channels for which they are used are error free [10]. In practice this assumption is not true and therefore ECCs are often used as inner codes. In this case the black box containing them and the real channel could be approximated in some cases as errorless. But the ECCs are not constructed with the goal to fulfill the constraints of the channels. From this point of view it seems that the last encoder and the first decoder connected to the real channel with constraints (the inner code) should be the constrained code and not the ECC.

One approach to overcoming this problem with ordering the codes in cascade, at least for cascading RLL and ECCs, is to construct combined codes denoted as RLL-ECCs [11-23], which have RLL and error control properties simultaneously. Recently RLL-ECCs were obtained from 5G LDPC (low density parity check) codes, [11]. The question arose if a related construction technique could be applied also to other standardized codes, for example to some of the CCSDS codes in order to obtain RLL-ECCs. In this paper it is shown that it is possible, at least for three Convolutional codes specified by CCSDS for optical communications in [24]. The obtained RLL-ECCs could be of interest for practical applications in space exploration.

We describe the relevant CCSDS convolutional codes and introduce the method for obtaining RLL-ECCs and

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some comments on its advantages as well as the obtained RLL-ECCs from the three CCSDS codes are presented.

2 Convolutional codes for optical communications recommended by ccstds 142.0-b-1

We present the basic information about binary convolutional CCSDS codes [24] for optical communications. The mother code of these codes is given by its generators over GF(2) (finite field with two elements).

$$\begin{aligned}
 g_1(x) &= x^2 + 1, \\
 g_2(x) &= x^2 + x + 1, \\
 g_3(x) &= x^2 + x + 1.
 \end{aligned}
 \tag{1}$$

The corresponding encoder is depicted in Fig. 1. In one cycle the number of bits is $k = 1$ and $n = 3$ which are inputted and outputted respectively. Therefore, the code rate $R = k/n$ for this mother code is 1/3. Using the puncture patterns given in Tab. 1 rate 1/2 or rate 1/3 code could be obtained.

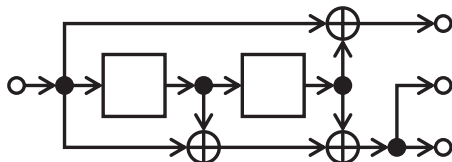


Fig. 1. The mother binary Convolutional Code recommended in[8] by CCSDS for optical communication systems

Table 1. Puncturing patterns for binary convolutional codes recommended in [24]

Code rate	Convolutional encoder puncturing patterns					
	P_0	P_1	P_2	P_3	P_4	P_5
1/3	1	1	1	1	1	1
1/2	1	1	0	1	1	0
2/3	1	1	0	0	1	0

3 RLL-ECCs obtained from convolutional codes for optical communications recommended by CCSDS 142.0-B-1

The method for obtaining an RLL-ECC from a binary ECC is illustrated in Fig. 2 for hard decoding and for transmission via BSC.

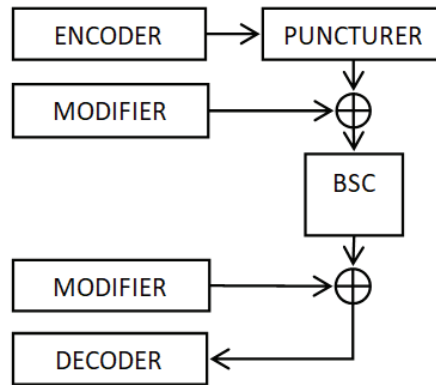


Fig. 2. The principle of how a binary ECC can be modified in order to get RLL properties without additional redundancy and without the need to change its encoding and hard decoding algorithms in BSC

The payload information is first encoded via the standard ECC into a codeword and punctured if required. In the following step some symbols are inverted by adding modulo two (mod 2) by a so-called modifier. For a convolutional code the modifier is a theoretically infinite binary sequence. On the receiving side, before the received symbols are inputted to the decoder, the modifier adds mod 2 to them in order to eliminate the influence on decoding from the operations made at the transmitting side.

The method illustrated in Fig. 2 has the following advantages:

- The problem with ordering ECC and translation codes mentioned in section 1 of this manuscript is overcome.
- No additional redundancy is introduced. The practical implementation has low complexity.
- The encoding and decoding of the original ECC does not have to be modified in cases where some round conditions are valid.
- The maximal run length L_{max} is guaranteed, which is not the case if scramblers are used.

The first three advantages are obvious. The 4-th advantage will also remain valid if soft decoding, coherent BPSK modulation is used and the channel can be modeled as an AWGN channel, [25].

The main problem which has to be solved before the method could be applied in practice is to find a modifier which will decrease the otherwise infinite run lengths in the codeword sequence of the original ECC. There are different approaches known regarding how to find a modifier[11-23] based on the structural properties of the original ECCs.

In this letter a simple greedy computerized search approach was used for obtaining RLL-ECCs from the convolutional codes standardized in[23]. The goal was to find positions of inversions in convolutional codewords which will decrease the run lengths of equal symbols in them. Observing the encoder in Fig. 1 the following Tab. 2 can be obtained, which contains in compact form the information which a codeword frame can follow after each codeword frame.

Table 2. Ordering of codeword frame in two consecutive time slots for mother binary convolutional code with coderate 1/3

Branch in time slot				
j	$j + 1$			
000	111	100	000	011
011	100	111	011	000
100	000	111	011	100
111	100	011	000	111

Table 3. Ordering of codeword frame in two consecutive time slots for mother binary convolutional code with coderate 1/3

Branch in time slot				
j	$j + 1$			
<u>010</u>	<u>101</u>	<u>110</u>	<u>010</u>	<u>001</u>
<u>000</u>	<u>111</u>	<u>100</u>	<u>001</u>	<u>010</u>
<u>110</u>	<u>010</u>	<u>101</u>	<u>001</u>	<u>110</u>
<u>101</u>	<u>110</u>	<u>001</u>	<u>010</u>	<u>101</u>

Table 4. Obtained results for the selected convolutional codes for TM space data link protocol specified by CCSDS, according to [12]

CCSDS code rate	L_{max}	Symbols inverted by modifier
1/3	3	2-nd symbol in every frame
1/2	8	1-st symbol in every second frame
2/3	12	1-st symbol in every fourth frame

After a search in the window containing two codeword frames it was found that it is possible to invert the second symbol in each frame by the addition of a modifier in order to use the method depicted in Fig. 2. After this modification, the information about consecutive branches is presented in Tab. 3, where the inverted values are underlined. Analyzing Tab. 3 it is obvious that after modification the mother convolutional code will have L_{max} .

For the two convolutional codes with coderate 1/2 and 2/3 obtained from the mother code with coderate 1/3 by the puncturing specified in [24] the computerized search was used in order to find the positions of inversions which would decrease L_{max} . The computerized search had two phases. In the first phase a search window was chosen, which contained all possible paths composed of 10 branches from all states. In other words, the search window L_{ws} was $10n_0$ bits long. Then for each path all possible modifiers were tried and the resulting sequence was analyzed in order to find L_{max} in it. For each path and modifier, the L_{max} was recorded. At the end of the first phase the candidate patterns were ordered from best (leading to a minimal value of L_{max}) to the worst. In the second phase the selected inversion patterns from the first phase were tested starting from the best one. In contrast to the first phase, now all paths containing 20 branches were tested after they had been modified by

the particular candidate inversion pattern. The resulting sequence was then analyzed on the maximal run-lengths which it contained. If the maximal run-length was higher than expected, the candidate inverting pattern was discarded and the next one was tested. This was done until some small value of L'_{max} was found. The corresponding modifier was also stored.

Note 1. It is obvious that the number N of these possible modifiers increases exponentially with the number of symbols in the window L_{ws} , namely $N = 2^{L_{ws}}$.

Therefore, the chosen value of L_{ws} in the described computerized search is restricted by the computing power which is at the disposal for running it.

Let us now concentrate our attention on the CCSDS convolutional code with coderate 1/2 obtained by puncturing each third symbol of each branch in the mother CCSDS convolutional code.

After the first computerized search phase it was found that inverting the first symbol in every second codeword frame will lead to $L_{max} = 8$. The verification in the second phase did not find any run length which was longer than 8.

The computerized search for a modifier with a small value of L_{max} for a CCSDS convolutional code for optical communications with coderate 2/3 revealed that inverting every first symbol in every fourth frame will lead to $L_{max} = 12$. The test in the second phase did not find any run length longer than 12.

The results are presented in Tab. 3. in a compact form. There is also information as to which symbols in which codeword frames of a concrete convolutional code have to be inverted by a modifier.

Note 2. The search revealed more different modifiers, which lead to the same value of L_{max} for convolutional codes with coderate 1/2 and 2/3. However, only one of them was selected for publication in Tab. 3 - specifically for each code which seemed to the authors to have the simplest implementation rules.

3 Conclusions

In this paper it was shown that RLL-ECC codes could be obtained from the convolutional codes specified by CCSDS in [24] using the method with modifiers. The main advantages of these codes are that the run lengths of equal symbols are restricted to corresponding values with a guarantee, that no additional redundancy has to be introduced in encoding, and that the encoding and decoding of the original error control codes specified by CCSDS do not have to be modified.

It is also possible that future research can bring new results with lower values of L_{max} , even for some of the three codes presented in this paper. The reason is that a, brute force or greedy search for modifiers minimizing L_{max} in convolutional codes specified by CCSDS can bring further progress if the search is made in larger search windows.

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