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New application of the key term separation principle

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The paper deals with a new application of the key term separation principle in identification of nonlinear dynamic systems. A multiplicative form of this operator decomposition technique is proposed and applied to the Wiener model. The resulting mathematical model is linear in both the linear and the nonlinear block parameters. Illustrative examples are included.

K e y w o r d s: nonlinear systems, identification, key term separation principle, Wiener model

1 Introductioon

The approaches dealing with nonlinear systems are generally restrictive in assumptions and applicable to special classes of systems only. In the case of nonlinear block-oriented systems with expected compound operators the main problem is caused by the mathematical tractability of descriptions given by the composition of mappings or operators $f = f_1 \, o \, f_2 \, o \ldots o \, f_n$, where f_i describes the behavior of ith block. These can be hardly used in their analytic form, and we are usually forced to use appropriate approximations and/or simplifications.

The decomposition technique based on the application of the so called "key term separation principle" offers an interesting possibility how to cope with such complex systems and was introduced by the author in [1]. The proposed technique enables to decompose a compound mapping leading to n equations exhibiting special qualities, which can be useful for iterative and recursive algorithms of identification and control, as well. In the simplest case of the Hammerstein and Wiener systems, where $f = f_1 \, o \, f_2$, the mathematical model consists of two equations and the inner mapping is included into the modified outer mapping both implicitly and explicitly.

In the previous works of the author the additive form of decomposition has led to new descriptions for the block oriented nonlinear models of Hammerstein and Wiener types with different types of both static [2] and dynamic nonlinearities [3] and the resulting model descriptions were nonlinear-in-variables, but linear-in-parameters and the identification problem turned to a quasi-linear one. Moreover, the key term separation principle was successfully applied in a large number of publications dealing with modeling and identification of nonlinear dynamic systems, see *eg* [4-12]. However, the key assumption in the case of Wiener models might be too restrictive in some cases.

In this paper a multiplicative form of compound operator decomposition is presented with the aim to overcome the mentioned problems. In the following a brief account is devoted to the above-mentioned decomposition technique and an appropriate form is introduced for the multiplicative case of decomposition. Then the proposed decomposition is applied to the Wiener model leading to a special form of model where both the linear and the nonlinear block descriptions appear in the resulting expression in unmodified form. Hence the decomposition is leading to a "parsimonious" model with the least possible number of parameters to be estimated. An iterative algorithm for estimation of the Wiener model parameters is proposed and its feasibility is illustrated by two examples.

2 Key term separation principle

Let f, g and h be mappings defined on nonempty sets U, X, and Y

$$f : U \to X, \quad g : X \to Y, \tag{1,2}$$

$$h = g \, o \, f : U \to Y, \tag{3}$$

ie for every $u \in U$, $x \in X$, $y \in Y$

$$y = g(x) = g[f(u)] = h(u), \tag{4}$$

$$x = f(u). \tag{5}$$

Assume the mapping g can be decomposed, *ie* splitted and uniquely replaced by two mappings

$$a : X \to Y, \quad b : X \to Y, \tag{6,7}$$

as ab. Then the mapping g can be defined on the Cartesian product of two identical copies of X,

$$g = ab : X \otimes X \to Y. \tag{8}$$

The replacement of the original domain set by the Cartesian product of the same sets is correct, it does not change the set topology and generally does not require any assumptions or restrictions. Now we can apply the

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decomposition (8) to the outer mapping g in the compound mapping (3) and the inner mapping f substitute explicitly only once. This gives

$$h = (a \circ f) \times b : U \otimes X \to Y, \quad (9)$$

where the domain of compound mapping is now the Cartesian product $U \otimes X$. Then (4) defining the given compound mapping can be rewritten as

$$y = g(x) = (a \circ b)(x) = a[f(u)] \circ b(x) = G(u, x), \quad (10)$$

and it will be valid on the sets U, X, and Y (or on the appropriate subsets of given mappings' domains). Hence the original compound mapping (3) can be replaced by the couple of mappings (1) and (9). It means that the original mapping defined by the relation (4) can be described by the couple of relations (5) and (10).

The choice of appropriate decomposition form for (8) can simplify in some cases the complex nonlinear mappings and descriptions of relations in (4). An important way of mapping decomposition is based on rewriting of mapping into an appropriate additive or multiplicative form, given by the sum or the product of at least two mappings. It means that for the mapping $g(.)$, with arguments from X there will be two mappings $a(.)$ and $b(.)$, such that in the *additive case*

$$g(x) = a(x) + b(x), \quad (11)$$

while in the *multiplicative case*

$$g(x) = a(x)b(x), \quad (12)$$

for any $x \in X$. Note that in the case of analytic mappings these decomposition forms always exist. Let the mapping g with domain X and range Y be decomposed into the multiplicative form (12) as

$$y = g(x) = \frac{c - x}{c - x} g(x), \quad (13)$$

with appropriately chosen c. In the simplest case we can choose $c = 1$. Rearranging (13) gives

$$y = xy + g(x)(1 - x). \quad (14)$$

The half-substitution of (5) into the right-hand side of (14), *ie* , only for x in the first term, leads to the expression

$$y = f(u)y + g(x)(1 - x), \quad (15)$$

where both $f(u)$ and $g(x)$ appear explicitly. The original mapping g, defined by (4), is now described equivalently by two expressions, *ie* , (5) and (15), which are peculiar to the intent that both expressions contain the same term on the right-hand side, namely $f(u)$. It means that the inner mapping appears in the outer one both implicitly and explicitly.

3 Wiener model

The Wiener model is given by the cascade connection of a linear dynamic system followed by a static nonlinearity block. The difference equation model of its linear dynamic block can be given as

$$x(t) = A(q)u(t) + [1 - B(q)]x(t), \quad (16)$$

where $u(t)$ and $x(t)$ are the inputs and outputs, respectively, $A(q)$ and $B(q)$ are scalar polynomials in the unit delay operator q^{-1}

$$A(q) = a_0 + a_1 q^{-1} + \cdots + a_m q^{-m}$$
$$B(q) = 1 + b_1 q^{-1} + \cdots + n_m q^{-n}. \quad (17{,}18)$$

The nonlinear block can be described by the equation

$$y(t) = G[x(t)], \quad (19)$$

where $x(t)$ is the input, $y(t)$ is the corresponding output. Hence the Wiener model is characterized by a compound mapping from the set of model inputs $u(t)$ into the set of model internal variables $x(t)$ and then into the set of model outputs $y(t)$.

Assume the nonlinear mapping $G(.)$ can be approximated by the polynomial of appropriate degree

$$y(t) = \sum_{k=1}^{r} g_k x^k(t). \quad (20)$$

After fixing $g_1 = 1$, (20) can be rewritten as

$$y(t) = x(t) + \sum_{k=2}^{r} g_k x^k(t). \quad (21)$$

Then choosing $x(t)$ as the key term and substituting (16) only for this separated $x(t)$, the model output will be

$$y(t) = \sum_{i=1}^{m} a_i\, u(t-i) - \sum_{j=1}^{n} b_j\, x(t-j) + \sum_{k=2}^{r} g_k\, x^k(t). \quad (22)$$

Equation (22) and that of (16) defining the internal variable $x(t)$ represent a special form of the Wiener model with polynomial nonlinearity where all the system parameters are given explicitly. The model parameter estimation can be performed iteratively with internal variable estimation [13], [14].

In the above case, the additive form of the key term separation principle has been used. However, the assumption on the description of nonlinear block might cause problems. Specifically, fixing the parameter value of the linear term in the polynomial approximation to one ($g_1 = 1$) could lead to an unwanted change of linear dynamic block gain. On the other hand, the model is not appropriate for polynomial characteristics with zero linear term.

This problem can be overcome by using a multiplicative form of the key term separation principle [15]. According to (12)-(14) the model output equation can be written as

$$y(t) = x(t)y(t) + G[x(t)][1 - x(t)]. \tag{23}$$

Now, after substituting (16) only for x(t) in the first term and considering more general output equation

$$y(t) = \sum_{k=0}^{r} g_k\, x^k(t), \tag{24}$$

we obtain

$$y(t) = \sum_{i=1}^{m} a_i\, u(t-i)y(t) - \sum_{j=1}^{n} b_j\, x(t-j)y(t) + \sum_{k=0}^{r} g_k\, x^k(t)[1 - x(t)]. \tag{25}$$

where all the parameters of both original mappings (16) and (24) appear . The resulting equation (25) is linear in all the Wiener model parameters; hence the parameter estimation problem can be solved as quasi-linear one with internal variable $x(t)$ estimation. It is important, that no restrictions are imposed on the model nonlinear block parameters.

4 Estimation algorithm

As the internal variable $x(t)$ in (25) is unmeasurable, the model parameter estimation can be performed using the iterative technique with internal variable estimation. The Wiener model given by (25) can be put into a concise form

$$y(t) = \Phi^\top(t)\Theta, \tag{26}$$

where the data vector is defined as

$$\Phi^\top(t) = \big\{u(t-1)y(t), \dots, u(t-m)y(t), -x(t-1)y(t), \dots, -x(t-n)y(t),\ [1-x(t)], x(t)[1-x(t)], \dots, x^r(t)[1-x(t)]\big\}, \tag{27}$$

and the vector of parameters is

$$\Theta^\top = [a_1, \dots, a_m, b_n, \dots, b_n, g_0, g_1, \dots, g_r]\,. \tag{28}$$

As a fact, no one-shot estimation algorithm can be applied to (26) because $\Phi(t)$ depends on an unmeasurable variable.

The proposed iterative algorithm is based on the use of the preceding estimates of model parameters for the estimation of internal variable. Assigning the estimated variable in the s-th step as

$${}^s x(t) = \sum_{i=1}^{m} {}^s a_i\, u(t-i) - \sum_{j=1}^{m} {}^s b_i^s x(t-j), \tag{29}$$

the error to be minimized is gained from (26) in the vector form

$$e(t) = y(t) -^s \Phi^\top(t)\, {}^{s+1}\Theta, \tag{30}$$

where ${}^s\Phi(t)$ is the data vector with the corresponding estimates of internal variable according to (29) and ${}^{s+1}\Theta$ is the $(s+1)$-estimate of the parameter vector.

The steps in the iterative procedure may be now stated as follows

Step 1: The initial estimates are made only for the parameters of linear block and used in (29) for the initial estimates of internal variable.

Step 2: Minimizing an appropriate criterion (*eg* least squares) based on (30) the estimates of both linear and nonlinear block parameters ${}^{s+1}\Theta$ are obtained using ${}^s\Phi(t)$ with the s-the estimates of internal variable.

Step 3: Using (29) the estimates of ${}^{s+1}x(t)$ are evaluated by means of the recent estimates of model parameters.

Step 4: If the estimation criterion is met the procedure ends, else it continues by repeating steps 2 and 3.

Note that because of the cascade connection of two blocks, their parameterization is not unique, as many combinations of parameters can be found. Therefore one parameter has to be fixed and we can assume that $a_1 = 1$.

5 Illustrative examples

Several Wiener systems were simulated and the estimations of all the model parameters were carried out on the basis of input and output records as well as the estimated internal variables. The performance of the proposed method is illustrated on the following examples.

Example 1.

The linear dynamic block of the Wiener system was described by the equation

$$x(t) = u(t-1) - 0.6u(t-2) + 0.5u(t-3) - 0.65x(t-1) - 0.35x(t-2)$$

and the nonlinear block (Fig. 1) was given as

$$y(t) = 0.2 + 0.8x(t) + 0.4x^2(t) + 0.1x^3(t)$$

The identification was performed on the basis of 3000 samples of uniformly distributed random inputs with $|u(t)| < 0.7$ and simulated outputs. Normally distributed random noise with zero mean and signal-to-noise ratio - $SNR = 50$ (the square root of the ratio of output and noise variances) was added to the outputs to make the

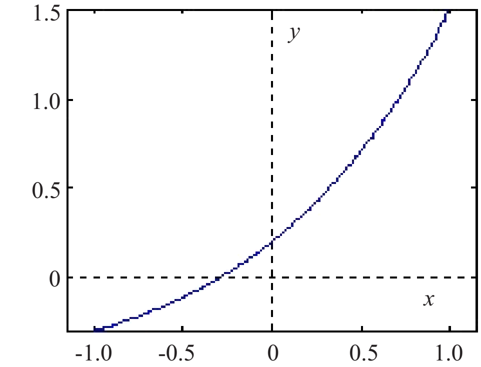

Fig. 1. Nonlinearity – Example 1

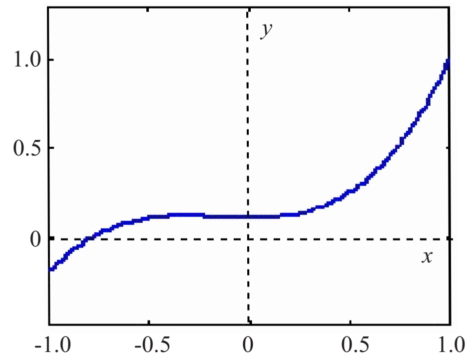

Fig. 3. Nonlinearity – Example 2

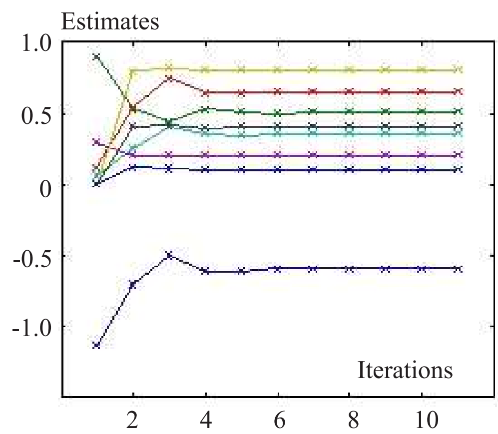

Fig. 2. Process of parameter estimation – Example 1

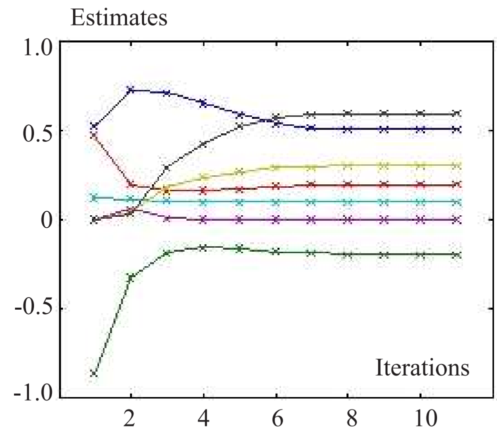

Fig. 4. Process of parameter estimation – Example 2

simulation more realistic. The process of parameter estimation is shown in Fig. 2 (the top-down order of parameters is: g_1, b_1, a_3, g_2, b_2, g_0, g_3, a_2). The estimates meet the values of real parameters after about 6 iterations.

Example 2.

The linear dynamic block of the Wiener system was described by the equation

$$x(t) = u(t-1) + 0.5u(t-2) + 0.2x(t-1) - 0.2x(t-2)$$

and the nonlinear block (Fig. 3) was given as

$$y(t) = 0.1 + 0.3x^2(t) + 0.6x^3(t)$$

The identification was performed on the basis of 3000 samples of uniformly distributed random inputs with $|u(t)| < 1$ and simulated outputs. Normally distributed random noise with zero mean and signal-to-noise ratio - $SNR = 50$ was added to the outputs to make the simulation more realistic. The process of parameter estimation is shown in Fig. 4 (the top-down order of parameters is: g_3, a_2, g_2, b_2, g_0, g_1, b_1). The estimates meet the values of real parameters after about 8 iterations.

Although there exists no exact proof of convergence for the utilized iterative method with the internal variable estimation, the tests exhibited good convergence. The required accuracy of identification was reached after about 6-8 iterations.

6 Conclusions

The additive case of the key term separation principle previously used for compound mapping decomposition has been extended to the multiplicative one. The proposed arrangement of the outer mapping and the following half-substitution of the inner one has led to such a description of the compound mapping where both mappings appear in the original form. This has been applied to the Wiener model and provided an output equation, which is linear in all the model parameters. Hence the Wiener model parameter estimation can be performed iteratively with internal variable estimation.

Finally note that the proposed approach can be used for different types of nonlinearities, both static (discontinuous, two segment polynomial, piece-wise linear) and dynamic (backlash, hysteresis).

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