

# MULTI-POSITION STATIC TEST OF MAGNETOMETER FROM IMU

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The article deals with so called multi-position static tests of magnetometer. This method is commonly used for Inertial Measurement Unit (IMU) accelerometer calibration. The goal of these tests is to approximate static conversion characteristics of 3-axis IMU magnetometer and to determine additive and multiplicative constants for x, y, z channels of the magnetometer.

Keywords: magnetometer, calibration, multi-position static test, additive constant, multiplicative constant

## 1 INTRODUCTION

Magnetometric sensors became a part of inertial navigation systems (INS). Their necessity very often results from the need to eliminate time dependent cumulative errors of inertial sensors. Magnetometers also allow to integrate the classic methods of inertial navigation with techniques based on geophysical fields very efficiently. The article deals with the method of magnetometric sensor calibration called multi-position static tests (MST) commonly used for INS inertial sensors calibration [1]. The goal of these tests is to approximate static conversion characteristics of 3-axis magnetometer based on analysis of measurement results obtained by rotation of the magnetometer from IMU placed on turning platform in the vertical plane around  $x$ ,  $y$  and then  $z$  - axis.

## 2 THEORY

Output data of the IMU sensors are in a certain measure influenced by noise and interference. Signal integration with its error components and with the following use in navigation algorithms leads to the significant error in position and velocity determination. Successful design, construction and expected parameters then depend on the thorough knowledge about used sensors and signal-processing systems.

In case of the approximation of the 3-axis magnetometer static conversion characteristic, it is important to determine additive and multiplicative constants, linearity and orthogonality errors.

Determination of conversion characteristic parameters by positioning of the sensor in and against the direction of the local magnetic field doesn't allow the calculation of non-linearity and misalignment. Therefore it is more purposeful to realise MST based on analysis of results obtained by sequential rotation around 3 axis ( $x$ ,  $y$  and  $z$ ) of the magnetometer positioned in the horizontal plane. If the orthogonality is good, then the rotation around  $x$ -axis positioned in the horizontal plane causes  $y$  and  $z$ -axis to rotate in the vertical plane and so characteristics of their output signals are ideally sinusoids with the same amplitude, zero offset and with the phase

shift of  $90^\circ$ . But the reality is different and mathematically we can write the relationship

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} m_{xx} & m_{xy} & m_{xz} \\ m_{yx} & m_{yy} & m_{yz} \\ m_{zx} & m_{zy} & m_{zz} \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} + \begin{bmatrix} b_{x0} \\ b_{y0} \\ b_{z0} \end{bmatrix} \quad (1)$$

where values  $b_x$ ,  $b_y$ ,  $b_z$  are uncalibrated output signals of the sensor,  $m_{xx}$ ,  $m_{yy}$ ,  $m_{zz}$  are multiplicative (sensitivity) constants,  $b_{x0}$ ,  $b_{y0}$ ,  $b_{z0}$  are additive constants,  $m_{xy}$ ,  $m_{xz}$ ,  $m_{yx}$ ,  $m_{yz}$ ,  $m_{zx}$ ,  $m_{zy}$  are orthogonality constants and  $B_x$ ,  $B_y$ ,  $B_z$  are calculated calibrated magnetometer outputs. These components of the local magnetic field in the area of measurement have to fulfil the condition:

$$B_x^2 + B_y^2 + B_z^2 = B^2 \quad (2)$$

Six magnetometer positions on the turning platform allow to write 3 x 6 equations that are necessary for calculation of the additive constant matrix  $b_0$  and multiplicative constant matrix  $m$ . The general rotation matrix  $R$  is given by the sequence of three rotations around each axis  $x$ ,  $y$ ,  $z$ , respectively [2]

$$\begin{aligned} R_x(\psi) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}, R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \\ R_z(\varphi) &= \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3) \\ R &= R_z(\varphi)R_y(\theta)R_x(\psi) \quad (4) \end{aligned}$$

For the next calculations we will use designations  $b_{xxi}$ ,  $b_{xyi}$ ,  $b_{xzi}$  for sampled data of sensor output signals by rotation around  $x$ -axis,  $b_{xyi}$ ,  $b_{yyi}$ ,  $b_{zyi}$ , by rotation around  $y$ -axis and  $b_{xzi}$ ,  $b_{yzi}$ ,  $b_{zzi}$  by rotation around  $z$ -axis. On the basis of the physical reality for the real output signals we can write

$$\begin{aligned} b_{xx} &= b_{xxA} \sin(\psi + \psi_{xx0}) + b_{xx0}, & b_{xy} &= b_{xyA} \sin(\psi + \psi_{xy0}) + b_{xy0} \\ b_{xz} &= b_{xzA} \sin(\psi + \psi_{xz0}) + b_{xz0}, & b_{yz} &= b_{yzA} \sin(\theta + \theta_{yz0}) + b_{yz0} \end{aligned}$$

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$$\begin{aligned}
 b_{yy} &= b_{yyA} \sin(\theta + \theta_{yy0}) + b_{yy0}, & b_{yz} &= b_{yzA} \sin(\theta + \theta_{yz0}) + b_{yz0} \\
 b_{yz} &= b_{yzA} \sin(\theta + \theta_{yz0}) + b_{yz0}, & b_{zx} &= b_{zxA} \sin(\phi + \phi_{zx0}) + b_{zx0} \\
 b_{zy} &= b_{zyA} \sin(\phi + \phi_{zy0}) + b_{zy0}, & b_{zz} &= b_{zxA} \sin(\phi + \phi_{zx0}) + b_{zz0}
 \end{aligned} \quad (5)$$

Values  $b_{xxA}$ ,  $b_{xyA}$ , ...  $b_{zxA}$  represent amplitudes,  $b_{xx0}$ ,  $b_{xy0}$ , ...  $b_{z0}$  are unidirectional shifts of corresponding sinusoidal characteristics and angles  $\psi_{xx0}$ ,  $\psi_{xy0}$ , ...  $\varphi_{zz0}$  are corresponding to initial phases. It is possible to calculate these coefficients from sampled characteristics  $b_{xxi}$ ,  $b_{xyi}$ , ...  $b_{zxi}$  and  $b_{xxA}$ ,  $b_{xyA}$ , ...  $b_{zxA}$ . For rotation around  $x$ -axis in the range of  $360^\circ$  with the step of  $1^\circ$  we can write equations:

$$b_{xx0} = \frac{1}{N} \sum_{i=1}^N b_{xxi}, \quad b_{yx0} = \frac{1}{N} \sum_{i=1}^N b_{yxi}, \quad b_{zx0} = \frac{1}{N} \sum_{i=1}^N b_{zxi} \quad (6)$$

$$\begin{aligned}
 b_{xxA} &= \sqrt{\frac{1}{N} \sum_{i=1}^N (b_{xxi} - b_{xx0})^2}, & b_{yxA} &= \sqrt{\frac{1}{N} \sum_{i=1}^N (b_{yxi} - b_{yx0})^2} \quad (7) \\
 b_{zxA} &= \sqrt{\frac{1}{N} \sum_{i=1}^N (b_{zxi} - b_{zx0})^2},
 \end{aligned}$$

where  $N = 360$  and similarly we can write equations for rotations around  $y$  and  $z$ -axis.

The additive constant of the sensor in direction of one of axes is calculated as an arithmetic average of unidirectional signal shifts of this channel by rotations around all 3 axes. For  $b_{x0}$ ,  $b_{y0}$ ,  $b_{z0}$  then we can write

$$\begin{aligned}
 b_{x0} &= -\frac{1}{2}(b_{xy0} + b_{xz0}), & b_{y0} &= -\frac{1}{2}(b_{yx0} + b_{yz0}) \\
 b_{z0} &= -\frac{1}{2}(b_{zx0} + b_{zy0})
 \end{aligned} \quad (8)$$

Multiplicative constants can be determined after calculation of signal amplitudes  $b_{xxA}$ ,  $b_{xyA}$ , ...  $b_{zxA}$  of the particular axis as the arithmetic averages of amplitudes, which correspondent with particular rotations. Constants are then normalized according to channel with the lowest sensitivity

$$A_{xx} = \frac{1}{2}(b_{xyA} + b_{xzA}), \quad A_{yy} = \frac{1}{2}(b_{yxA} + b_{yzA}), \quad A_{zz} = \frac{1}{2}(b_{zxA} + b_{zyA}) \quad (9)$$

$$A_{\min} = \min \langle A_{xx}, A_{yy}, A_{zz} \rangle \quad (10)$$

$$m_{xx} = \frac{A_{\min}}{A_{xx}}, \quad m_{yy} = \frac{A_{\min}}{A_{yy}}, \quad m_{zz} = \frac{A_{\min}}{A_{zz}} \quad (11)$$

### 3 EXPERIMENTS

Constant and homogeneous magnetic field is the essential condition for magnetometer calibration. We monitored the time/space motion of magnetic background in the testing area by one-axis vector magnetometer Vema-030 [3], [4] and 3-axis magnetometer HMR-2300.

The level of the magnetic field in time, its spatial distribution and amplitude of the noise in the measurement area confirmed that magnetic field was constant and homogeneous with no significant changes in nT range and with minimal interference (Fig. 1.).

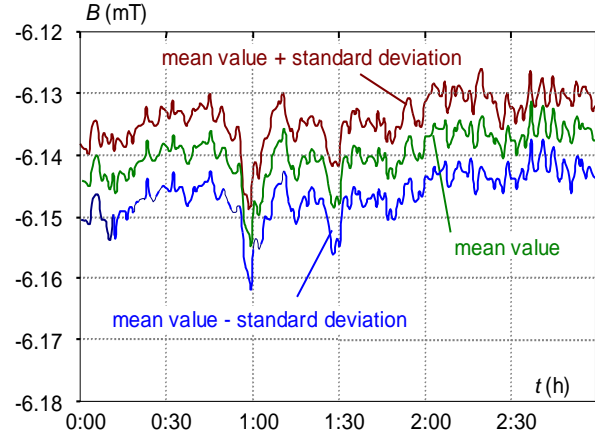


Fig. 1. Magnetic field time changes in the testing area

Measurements were realized with the 3-axis magnetometer HMC2003 manufactured by Honeywell, which is a part of the custom MEMS-based IMU with 9 degrees of freedom (DOF). For comparison there were also measurements performed on commonly used commercial low-cost 3-axis magnetic sensor module MicroMag.

Since magnetometers used in tests are IMU components, by measurements we used a MST methodology usually applied to accelerometer calibration. Magnetometer was situated either in the plane of acceleration of gravity  $g$  or in the plane perpendicular to  $g$  vector in the Cartesian coordinate system. The MST was performed as followed: IMU was rotating in each plane using turning platform with step of  $1^\circ$  in the range of  $450^\circ$ , for each degree and each channel 100 samples were collected (Fig. 2.).

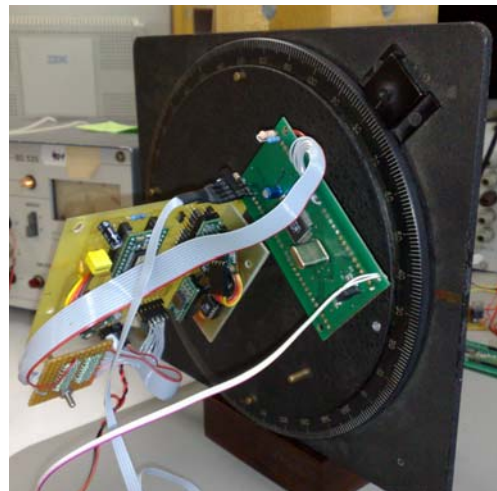
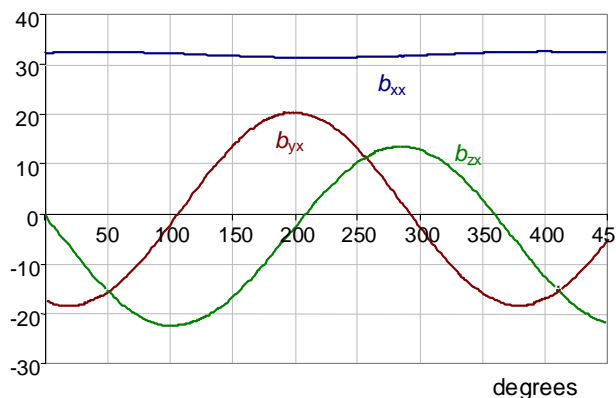


Fig. 2. Turning platform for IMU magnetometer testing



**Fig. 3.** Rotation around  $x$ -axis of HMC2003

Sampled data were evaluated for each plane separately; particular additive and multiplicative constants were calculated according to the equations 6, 7. Additive and multiplicative constants for 3D space were calculated from particular constants of each plane using (8) to (11).

Figures 3 to 6 illustrate the data evaluation and particular constants determination for one of three evaluated planes of sampled signals of HMC2003 magnetometer obtained by rotation around  $x$ -axis in  $yz$  plane.

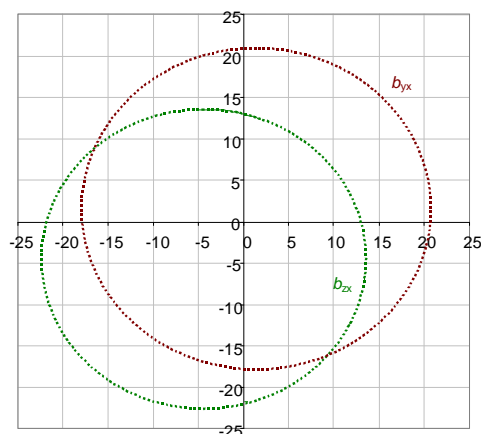
Figure 3 shows the sampled data for each channel and for each step of rotation. It is obvious that shifts and amplitudes in particular channel are very different.

For a better graphic visualisation of signal characteristics we can use Lissajous figure made from primary sampled signal and the same signal with the phase shift of  $90^\circ$ . For  $yz$  plane we need to evaluate signals  $b_{xy}$  and  $b_{xz}$ . Ideally, with zero shift and the same amplitude of  $b_{xy}$  and  $b_{xz}$ , we get two concentric overlapping circles.

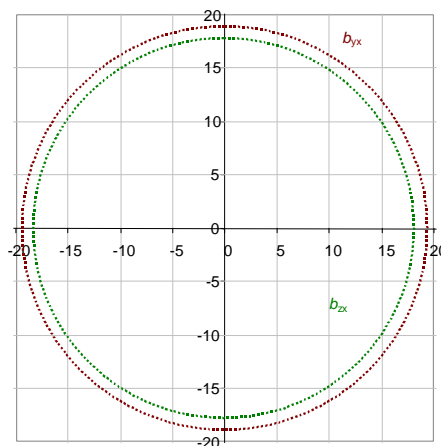
Figure 4 shows the real case of output signals  $b_{xy}$  and  $b_{xz}$ . Figure 5 shows circles after compensation of unidirectional shift using particular additive constants  $b_{yx0}$  and  $b_{zx0}$  for  $yz$  plane. Figure 6. shows circles from Figure 5 adjusted to the channel with lower sensitivity using particular multiplicative constants  $b_{yxA}$ ,  $b_{zxA}$  for  $yz$  plane.

Deformation of the circle is caused by non-linearity of the sensor conversion characteristics and it can be calculated as a maximum deviation from ideal circle.

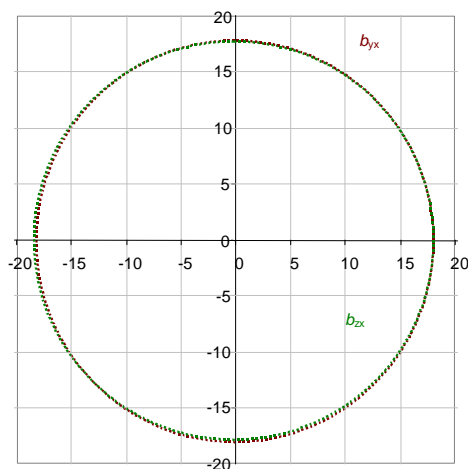
Partial additive and multiplicative constants for rotation around  $x$ ,  $y$  and  $z$ -axis were calculated using equations 1 – 11 for each plane. Partial results were used for determination of final additive constants  $b_{x0}$ ,  $b_{y0}$ ,  $b_{z0}$  and multiplicative constants  $m_{xx}$ ,  $m_{yy}$ ,  $m_{zz}$  for 3D space. Multiplicative constants are used for sensitivity normalization what means that we have exact information only about the direction of the vector  $B$  but for navigation purposes it is sufficient. For the exact information



**Fig. 4.** Visualisation of measured data units in both directions are  $\mu\text{T}$



**Fig. 5.** Visualisation of measured data after unidirectional shift compensation, units in both directions are  $\mu\text{T}$



**Fig. 6.** Visualisation of measured data after calibration using additive and multiplicative constants, units in both directions are  $\mu\text{T}$

about the magnitude of  $B$  correction according to the reference value would be necessary.

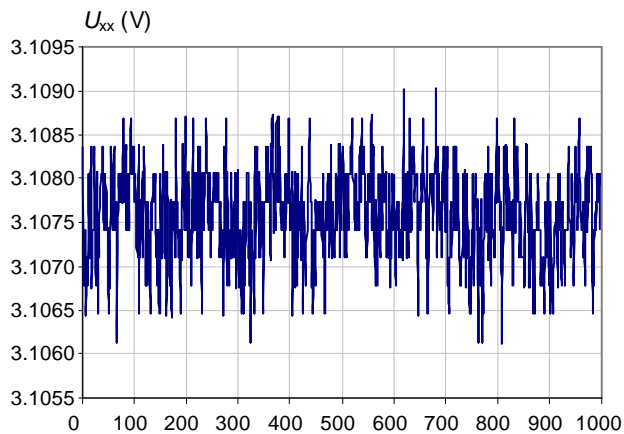


Fig. 7. Representative characteristic of measured signals

Results of data analysis, that we obtained by application of the MST method for HMC2003 and MicroMag magnetometers tests (Table 1., Table 2.) show that there is a significant variance of calculated additive and multiplicative constants of each magnetometer. Similar results are also common for magnetometers of the same type.

Table 1. Calculated additive constants

|          | $b_{x0}$ ( $\mu\text{T}$ ) | $b_{y0}$ ( $\mu\text{T}$ ) | $b_{z0}$ ( $\mu\text{T}$ ) |
|----------|----------------------------|----------------------------|----------------------------|
| HMC2003  | -13.39                     | -1.92                      | -1.55                      |
| MicroMag | 1.92                       | -1.56                      | -0.38                      |

Table 2. Calculated multiplicative constants

|          | $m_{xx}$ | $m_{yy}$ | $m_{zz}$ |
|----------|----------|----------|----------|
| HMC2003  | 1.00     | 0.48     | 0.61     |
| MicroMag | 0.94     | 1.00     | 0.89     |

The important factor that influences the precision of additive and multiplicative coefficients as well as orthogonality and non-linearity coefficients determination is noise inherent in sampled characteristics. Possibilities of the analysis are dependent on the used sampling frequency. Sampling frequency during measurements with HMC2003 was 1000 Hz while using DAQ NI WLS-9163 or 1036 Hz with ADuC812 microcontroller and during measurements with MicroMag it was only 17 Hz.

Conventionally used characteristic in data analysis is the variance  $\sigma^2$ , which is mainly used for sampled signal with constant mean value and normal distribution. Often used characteristic of inertial sensor output signal is Allan variance:

$$\sigma_A^2 = \sum (x_{k+1} - x_k)^2 / 2(n-1) \quad (12)$$

Nowadays it is often used in data processing and noise analysis. By MST we used Allan variance for number of samples determination, which are necessary for averaging, so that the signal variance caused by noise was constant. The number of samples for both magnetometers

was calculated to be more than 46, we used 100 samples. Noise amplitude while measuring with HMC2003 was  $0.042096 \mu\text{T}$  and with MicroMag  $0.406453 \mu\text{T}$ . Typical characteristic of measured signals is in Fig. 7 and Allan variance development in Fig. 8.

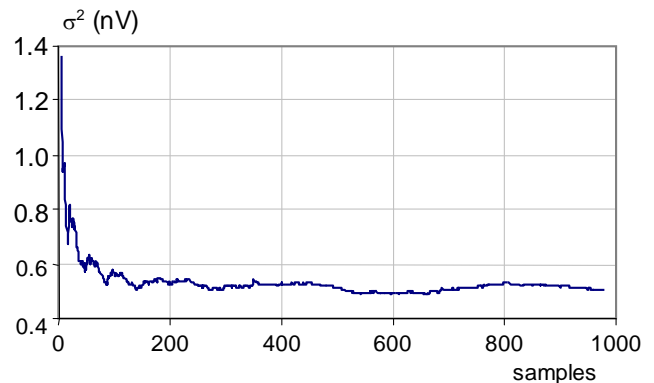


Fig. 8. Allan variance development

## 4 CONCLUSION

The presented MST method used for additive and multiplicative constants calculations offers a different view about the qualitative magnetometric sensor analysis. The graphic interpretation of measured data allows comparison of the particular magnetometer channels. Ideally we get concentric circles with the centre in the coordinate system origin. However the real graphic characteristics, especially of the magnetometric sensors, are often deformed as a result of the presence of soft and hard magnetic materials near by the sensor and also because of the effect of parasitic magnetic fields.

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## REFERENCES

- [1] ARTESE, G. – TRECROCI, A.: Calibration of a low cost MEMS INS sensor for an integrated navigation system. The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences. Vol. XXXVII. Part B5. Beijing 2008, 877-882.
- [2] SOTAK, M. – SOPATA, M. – BREDÁ, R. – ROHÁČ, J. – VACÍ, L.: Navigation System Integration. Monograph: 1. ed., printed by Robert Breda, Kosice, Slovak Republic, 2006, ISBN 80-969619-9-3.
- [3] BLÁZEK, J. – HUDÁK, J. – PRASLICKÁ, D.: A relax-type magnetic fluxgate sensor. In: Sensors and Actuators A: Physical. Vol. 59, No [1-3], 1997, 287-291, ISSN 0924-4247.
- [4] PRASLICKÁ, D.: A Relax-Type Magnetometer Using Amorphous Ribbon Core. IEEE Transactions on Magnetics, 30/2, 1994, 934-935, ISSN 0018-9464

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