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SIMPLIFIED DETERMINATION OF FLAT COIL FIELDS THROUGH ENERGY EQUIVALENCE

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The authors consider the possibility to approximate the field of a flat circular coil by the field of one or two turns. To this end, they define a general rule of equivalence between two sources of magnetic field. The application of this rule in two different manners gives encouraging results which are discussed in detail.

Keywords: flat coils, simplified analysis, magnetic energy, field equivalence

1 INTRODUCTION

In many cases, it is necessary to evaluate a magnetic field rapidly. Then, high accuracy is less important than rapidity. The object of the present paper is to give a method to replace a given field source by an "equivalent one" whose field is much easier to evaluate than the field of the original one, at the cost of a reasonable loss of accuracy. An efficient way of defining equivalence is through energy equality in a uniform field. We give below the example of a flat coil, and compare its field with the field of one or two well chosen turns. The pros and cons of the method will appear immediately.

2 FIELD AND ENERGY EQUIVALENCE OF TWO COILS

Consider two coils (1) and (2) as we shall do below. We shall say that they are "equivalent" if, for a given application, coil (2) has a field approximately equal to the field of (1), at least in a region of space which is of interest to us. If they are "equivalent" in that sense, their fields $(\vec{H}_1, \vec{B}_1)$ and $(\vec{H}_2, \vec{B}_2)$ will be close to each other at every point of space, so that their electromagnetic energies $\int \vec{H}_1 \cdot \vec{B}_1 dv$ and $\int \vec{H}_2 \cdot \vec{B}_2 dv$ will be approximately equal. These energies may be difficult to evaluate, anyway. But an interesting fact is that, if those two fields are equivalent as defined above, they will have the same mutual energy with a third field ("test field"), which we may choose as simple as a uniform field. Then, the mutual energy between coil (1) and the test field is as simple as $I_1 \Phi_{1\,test}$ where $\Phi_{1\,test}$ is the flux of the test field through coil (1). Same note for coil (2).

Those definitions have been successfully used in references [1, 2].

3 DESCRIPTION OF THE FLAT COIL

The flat coil is shown in Fig. 1. It consists of N series connected turns; the inner and outer diameters are called $2r_1$ and $2r_2$ respectively. The current is I.

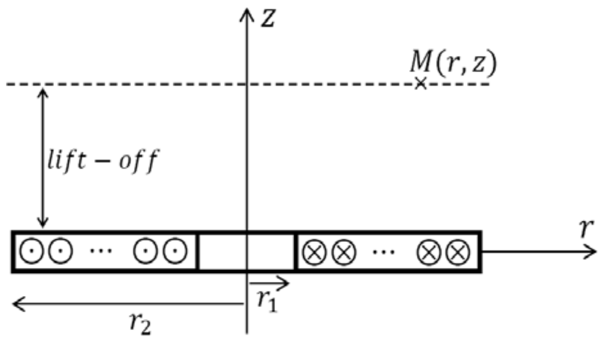

Fig. 1. Flat coil under consideration

4 EVALUATION OF THE FIELD

There exists a very complete analysis dating back to 1968 [3 - 6], which provides the value of the radial and axial components of the magnetic induction in air. The authors have established a partial differential equation which they have treated by variable separation, and use Bessel functions of the first kind. Easier procedures based on the use of Biot and Savart law, are also known [7].

5 ALLEVIATING THE COMPUTATIONAL BUREN

We may wonder if it is possible to replace the coil by only one turn, and obtain results which might be useful, at the cost of accuracy, at the place where this result is needed. How would we choose the unique turn?

The first answer which is offered is as follows. We first consider to replace the flat coil (N turns, current I by a unique turn, having the same symmetry plane and symmetry axis as the initial flat coil, and carrying a current equal to NI

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We have to choose then its radius. To this end, we shall require that the mutual energy W of the turn in a test field B_{test} be the same for the original coil and the equivalent turn. The simplest test field is a uniform field of magnetic flux density B_0 parallel to the axis of the flat coil (*cf* figure 2). Then, the energy W will result from

$$W = \pi B_0 I \sum_{1}^{N} r_i^2 = \pi B_0 I r_{equi}^2 , \quad (1)$$

where, r_1 is the radius of the i-th turn of the flat coil, and B_0 the density of uniform field B_{test}.

Therefore,

$$r_{equi}^2 = \frac{1}{3} \frac{r_2^3 - r_1^3}{r_2 - r_1} . \quad (2)$$

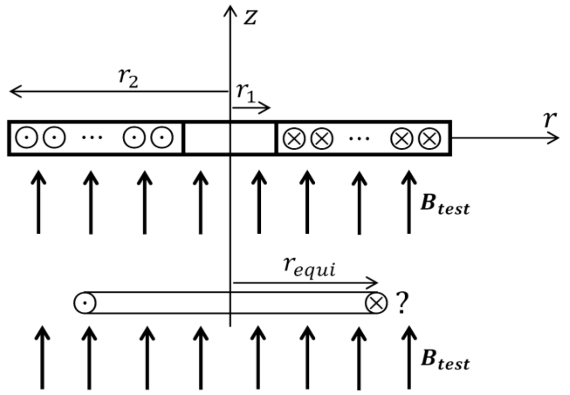


Fig. 2. Flat coil and equivalent turn under consideration

6 FIRST APPLICATION

We consider the case of $N = 50$, r_1=1 mm, r_2=20 mm, I = 1 A. We get r_{equi} = 11.27 mm, NI= 50 A. The field of a unique turn, of radius 11.27 mm, carrying 50 A, located in the plane of the actual coil, was computed in a plane distant of *lift-off* = 50 mm from the coil. Its value, as well as the values of its radial and axial components, are shown in Fig. 3(a).

Then, the same evaluations have been made for a plane distant of *lift-off* = 30 mm; the maximum error is 8%, *cf* Fig. 3(b).

7 SECOND APPLICATION

We may consider the same coil as above, dividing it in two concentric coils, one of inner and outer radius equal to r_1 = 1 mm and r_2 = 9.5 mm, the other one of inner and outer radius equal to r_1 = 9.5 mm and r_2 = 20 mm, both, having 25 turns. Two equivalent turns, each carrying 25 A, have radius equal to r_{equi} = 5.8 mm and r_{equi} = 15.7 mm, *cf* Fig. 4.

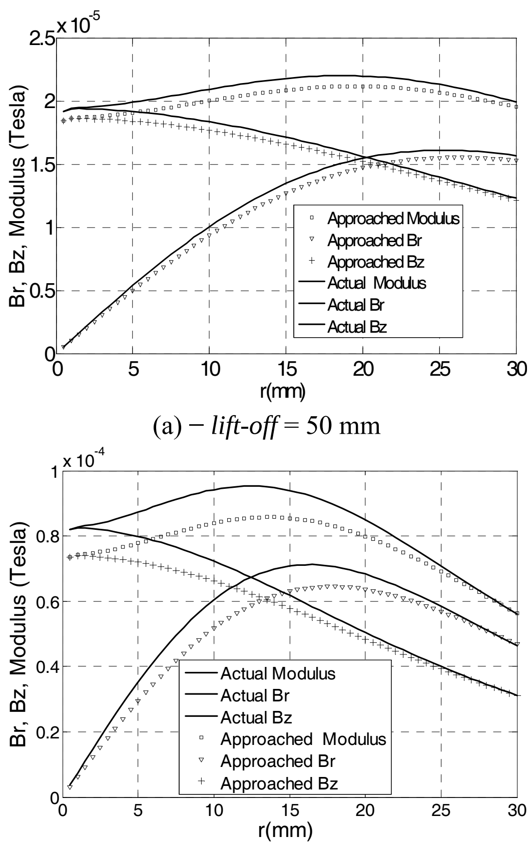


Fig. 3. Actual and approached values of the total field at two different *lift-off* , their radial and axial components, in the case of one equivalent turn

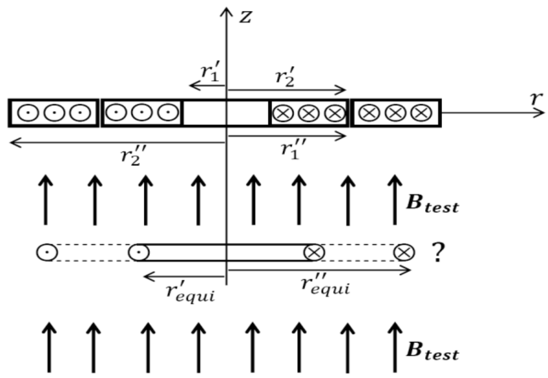
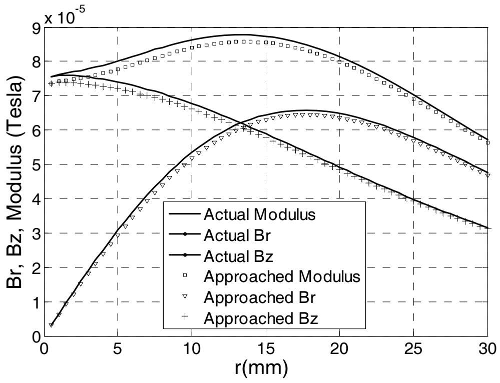
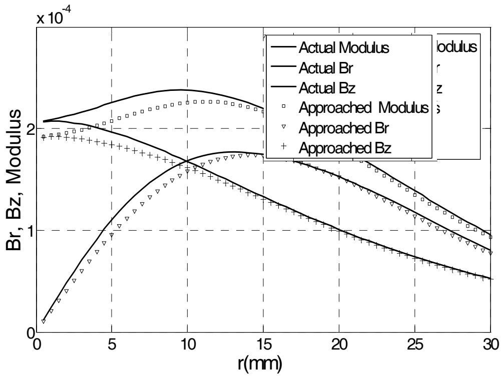


Fig. 4. Flat coil and equivalent pair of turns under consideration

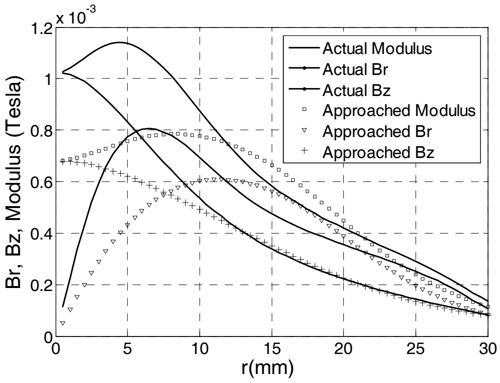
Adding their fields for *lift-off* = 30,20 and and 10 mm, we obtain the results displayed in Fig. 5(a), (b) and (c). Results are satisfactory for the two first ones, not for the third, as might be expected when we get closer to the coil. To improve the correlation, the initial coil can be divided in more than two concentric coils and so the corresponding equivalent turns, for smaller values of lift-off.



(a) − *lift-off* = 30 mm



(b) − *lift-off* = 20 mm



(c) − *lift-off* = 10 mm

Fig. 5. Actual and approached values of the total field at different *lift-off*, its radial and axial components, in the case of two equivalent turns

8 CONCLUSIONS

The idea of replacing a flat coil by one or two simple turns is quite natural. However, it is not obvious to choose those simple turns. That is why we give a simple rule of comparison of two different circuits immersed in a uniform "test field". We have used this rule to define first a simple turn, then a two turn circuit equivalent to the proposed flat coil. The result cannot be very satisfactory at an axial distance smaller than the outer radius of the coil. Applications have shown the pertinence of these definitions, and their inherent limits. Note that we have used the same principle for a simpler case in another publication [2], and there the result is perfect.

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