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ANALYTICAL MODEL WITH FLEXIBLE PARAMETERS FOR DYNAMIC HYSTERESIS LOOPS MODELLING

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Analytical model was proposed to find simple mathematical approximation of dynamic hysteresis loops of soft magnetic materials. It is based on pure phenomenological approach and consists in dynamic loop construction from four components by means of mathematical functions. Favourable mimicking coefficients were searched, which depend on excitation amplitude and frequency. The nature of its behaviour was studied in order to be able to predict the loop shape. The model was tested for opened specimen of classic oriented silicon electrical steel at sinusoidal excitation, measured by means of KF9a compensation ferrometer.

Keywords: dynamic hysteresis loop, mathematical modelling, analytical approximation

1 INTRODUCTION

Mathematical model based on hysteresis loop construction from analytical functions and suitable for AC magnetizing is proposed. It approximates symmetrical dynamic hysteresis loops and performs reasonably in the whole range of commonly used excitation.

It is a part of a more complex task – improving the properties of the measuring device KF9a [1] by means of simple prediction algorithm implementation. Knowing the rough estimate of material behaviour, the desired goal is optimum magnetizing parameters setting and measuring process accelerating.

In the first step, work was focused on suggestion of suitable analytical functions, which are able to approximate the shape of the loop successfully. After that, optimum coefficients finding and simulation quality verification follows. Further work consists of studying the coefficients behaviour with regard to excitation and frequency. Finally, the model should predict (for certain class of soft magnetic materials) the shape of dynamic loop and related important parameters.

2 METHODS USED FOR DYNAMIC HYSTERESIS LOOPS MODELLING AND ANALYTICAL FUNCTIONS UTILIZATION

Hysteresis loop modelling represents one of important ways of magnetic materials behaviour description. Hysteresis loops and other curves could be described by different mathematical and experimental models [2], [3].

Frequently used methods for dynamic loop modelling are represented by dynamic Preisach model and various differential models: Jiles-Atherton model, Chua model, Hodgdon model, Duhem model. There are a lot of other dynamic models available.

Most dynamic models give good results, however, for the purpose of KF9a prediction algorithm they seem to be rather complicated or time consuming. Hence more simple way was searched – analytical approximation.

Analytical approximation models represent the simplest way of describing magnetization curves by approximating their shape exploiting mathematical functions. Various convenient functions are used [2] – power series (*eg* Rayleigh), rational fractures (*eg* Frölich), exponential functions, tangent and inverse tangent functions; models of Takacs [4] (hyperbolic tangent), and Wlodarski [5] (Langevin function) should be mentioned as representatives of more recent approaches.

However, most of existing analytical models perform well at slow magnetizing only, with increasing frequency they are able to approximate saturated loops but fail in mimicking rounded smaller loops; or the full loop has to be put together from several segments.

3 PROPOSED ANALYTICAL METHOD FOR DYNAMIC HYSTERESIS LOOPS MODELLING

Proposed approach consists of dynamic loop composition from four components

$$B_{\pm} = g_1(H) + g_2(H) + g_3(H) + g_4(H). \qquad (1)$$

The B_+ and B_- terms denote ascending and descending branches of the loop, respectively. The upper sign in the symbols $\pm$, $\mp$ used below is valid for B_+ and the lower for B_-; H_a is applied field amplitude and H_c coercivity.

The model components (functions g_1, g_2, g_3, g_4) were derived simply from the knowledge of dynamic loop shape, apart from physical nature of dynamic magnetization mechanisms. They are defined as follows

$$g_1 = a_1 \frac{2}{\pi} \operatorname{sign}\left[\alpha\left(H \mp H_c\right)\right] \operatorname{atan}\left[\left|\alpha\left(H \mp H_c\right)\right|^{\frac{1}{2}}\right]. \qquad (2)$$

The g_1, based on inverse tangent [2], is responsible for the sigmoid shape of strongly saturated loop, whose shape is similar to the static one. Parameter α controls the slope of g_1. This component prevails at high saturation level (Fig.1), at low saturation g_1 is weak or disappears (Fig.2).

The next component represents contribution of the “lagged response” to the excitation signal

$$g_2 = \mp a_2 \left|1 - \left(\frac{H}{H_a}\right)^2\right|^{k_1}. \qquad (3)$$

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This g_2 component features the rounded as far as elliptical shape of dynamic loop at smaller saturation level (Fig. 2). Coefficient k_1 causes the necessary elliptical shape variation. In numerous cases the g_1 and g_2 components are sufficient for credible loop shape approximation. But sometimes correction is needed for the loop flexure

$$g_3 = \mp a_3 \left| k_2 \left(\frac{H}{H_c} \mp 1 \right) \right| \left(1 + \left[k_2 \left(\frac{H}{H_c} \mp 1 \right)^2 \right] \right)^{-1} . \qquad (4)$$

The g_3 component provides correction for greater loops, especially round H_c (Fig.1), but it is able to improve smaller loops approximation, too. In such case the midpoint is not H_c and it has to be shifted towards H_a

$$g_4 = a_4 H \pm b_0 . \qquad (5)$$

The g_4 component is linear, representing loop declination. For credible loop shape approximation only seven parameters (a_1, a_2, a_3, a_4, α, k_1, k_2) are required; b_0 is a constant, resulting from the condition

$$B_+(H_a) = B_-(H_a). \qquad (6)$$

In addition, in many cases, not all the model components are inevitable. The smallest of them can be neglected, so that the number of necessary parameters is less than seven.

The frequency and excitation dependence is hidden in the coefficients which have different values for each loop, so that it would be more correct to write $a_1(B_a, f)$, $\alpha(B_a, f)$ *etc*. However, the problem of variable parameters could be simplified when the coefficients dependence is to be revealed or when some coefficient stays constant.

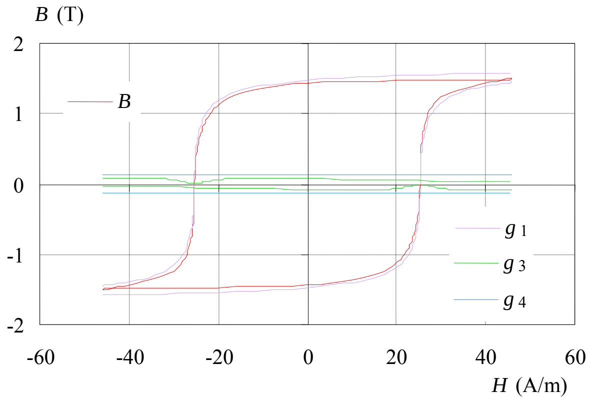

Fig. 1. Simulation for $f = 50$ Hz and $B_a = 1.5$ T

In Fig. 1, Fig. 2 examples are depicted of simulated hysteresis loops with separate components. Contribution of individual components depends on magnetic flux density amplitude B_a, final loops are depicted by red line. It is obvious from Fig. 1 that the elliptical g_2 component (*ie* coefficients a_2, k_1) can be omitted, and $a_4 = 0$. On the other hand, in Fig. 2, the g_3 component (*ie* coefficients a_3, k_2) can be ignored.

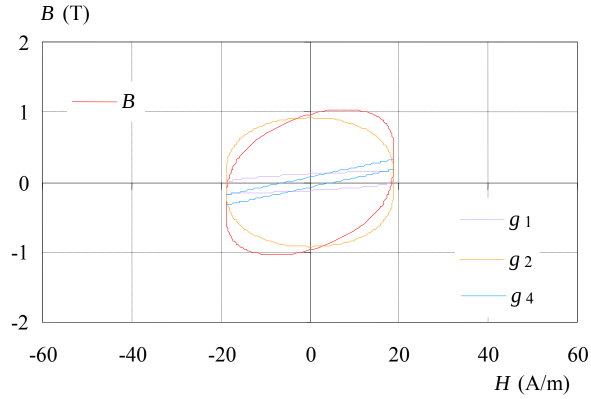

Fig. 2. Simulation for $f = 50$ Hz and $B_a = 1.0$ T

For appropriate parameters finding Newton method (for the first estimation) and then genetic algorithm were used, minimizing the mean squared deviation. Criteria of the least mean absolute deviation and least maximum absolute deviation were tested, too. The searching procedure was limited to non-negative parameter values.

For credible loop approximation, the fitness function could be improved by setting the condition of precise agreement in most important points of the loop.

4 TESTING OF PROPOSED MODEL AND RESULTS

Features of the proposed model were tested for classic soft magnetic material – oriented silicon electrical steel Eo10 (sheet 500 mm length, 500 mm width, thickness 0.35 mm) using sinusoidal magnetization at different frequencies f and magnetic flux densities B_a.

Material samples were measured in $N = 200$ points per loop by means of the compensation ferrometer KF9a, PC controlled single sheet/strip tester [1] at following conditions

$f = 50$ Hz, $\quad 0.1\ \text{T} \leq B_a \leq 1.7$ T (step 0.1 T);

$40\ \text{Hz} \leq f \leq 400$ Hz, $\quad B_a = 0.5$ T–1.0 T–1.5 T–1.7 T.

In the course of optimum parameters searching, the initial premise that model parameters are variables, dependent on saturation and frequency, proved true. Nevertheless, the parameters often stay rather invariable in certain range of saturation and frequency, or it is possible to anticipate their behaviour.

Experimental results demonstrated in Fig. 3 and Fig. 4 show rather good similarity with measured values (black line – measured loops, red line – simulated loops). Only in the end-points of several rounded loops appears discontinuity (see Fig. 4400 Hz).

Deviations between simulated and measured loops for both examples mentioned above are depicted in Fig.5 and Fig. 6. Results are expressed in %, related to measured B_a.

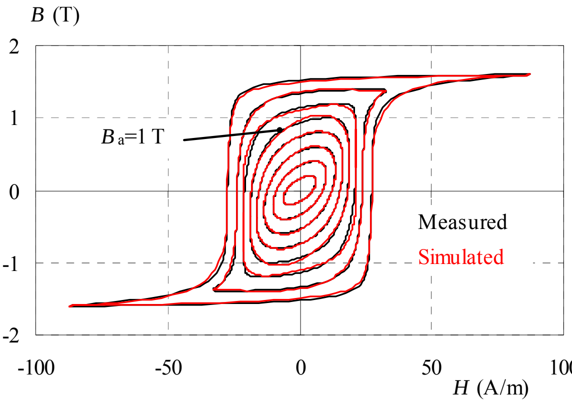

Fig. 3. Measured and simulated hysteresis loops at B_a = 0.2–0.4–0.6–0.8–1.0–1.2–1.4–1.6 T, f = 50 Hz

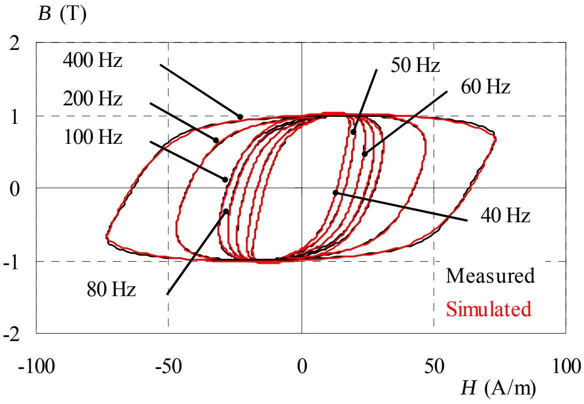

Fig. 4. Measured and simulated hysteresis loops at B_a = 1.0 T, f = 40–50–60–80–100–200–400 Hz

$$\delta = \frac{B_{\text{simulated}} - B_{\text{measured}}}{B_{a\ \text{measured}}} \cdot 100\,\% \tag{7}$$

In several points (Fig. 5) deviation seems rather high – it is due to high slope of hysteresis loop in the H_c area. However, the mean absolute deviation related to B_a stays less than 5 % for all the simulated loops.

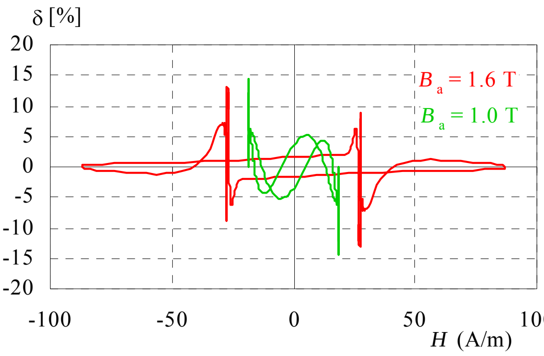

Fig. 5. Deviation between simulated and measured loops at f = 50 Hz, B_a = 1.0 T and 1.6 T

Results in Fig .6 are better, because respective loops have similar shapes (Fig. 4). The mean absolute deviation related to B_a stays less than 3 % for all simulated loops.

Parameter variations (of all the parameters a_1, a_2, a_3, a_4, α , k_1, k_2) at f = 50 Hz for increasing B_a are illustrated in Fig. 7 to Fig. 10, with related RMS deviation. In equation (8) N = 200 measured points.

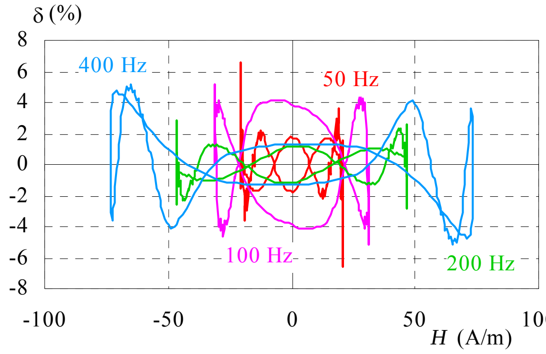

Fig. 6. Deviation between simulated and measured loops at f = 50 Hz – 100 Hz – 200 Hz – 400 Hz, B_a = 1.0 T

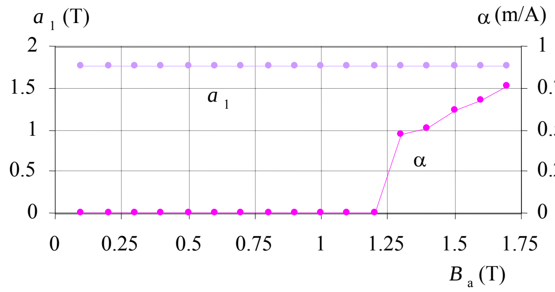

Fig. 7. Dependences of parameters a_1 (T) and α (m/A) at f = 50 Hz and increasing B_a

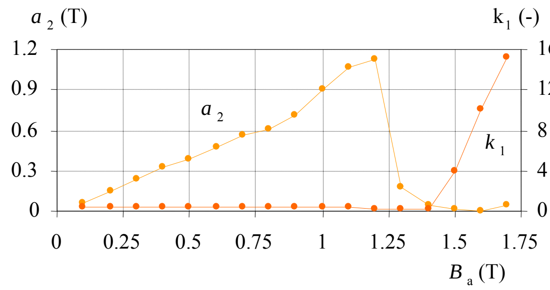

Fig. 8. Dependences of parameters a_2 (T) and k_1 (–) at f = 50 Hz and increasing B_a

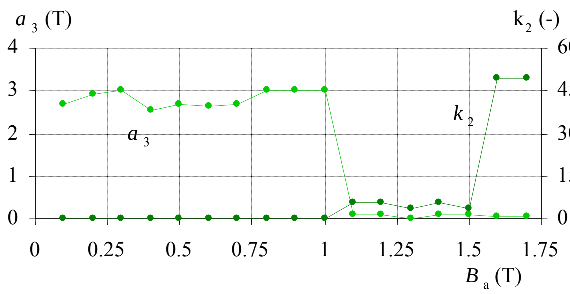

Fig. 9. Dependences of parameters a_3 (T) and k_2 (–) at f = 50 Hz and increasing B_a

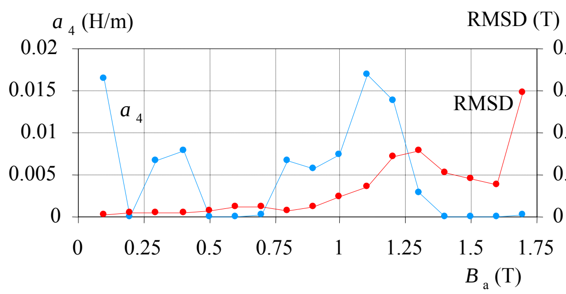

Fig. 10. Dependences of parameter a_4 (H/m) and RMS deviation (T) at f = 50 Hz and increasing B_a

$$\text{RMSD} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(B_{i\ \text{simulated}} - B_{i\ \text{measured}}\right)^2} \qquad (8)$$

Parameter variations at B_a = 0.5 T for increasing frequency are illustrated in Fig.11 – Fig.12, with related RMS deviation. In the latter case, parameters a_1, a_3, α , k_2 are not depicted because g_1, g_3 components were omitted. Nevertheless, the approximation quality is good, the mean absolute deviation related to B_a stays less than 4 %.

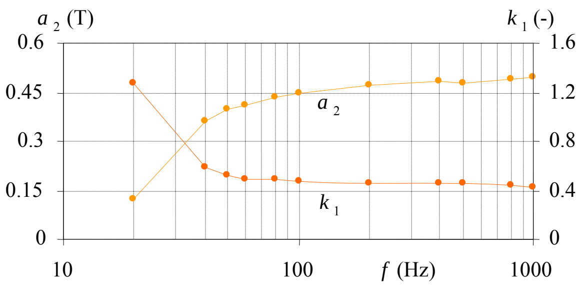

Fig. 11. Dependences of parameters a_2 (T) and k_1 (–) at B_a = 0.5 T and increasing f

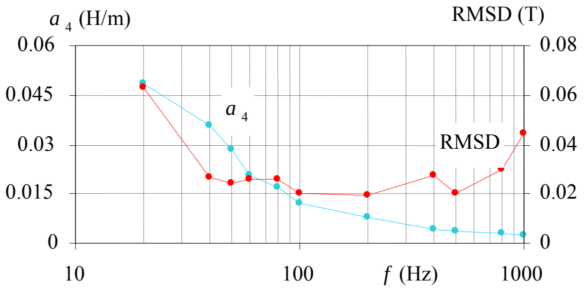

Fig. 12. Dependences of parameter a_4 (H/m) and RMS deviation (T) at B_a = 0.5 T and increasing f

It is obvious from Fig. 11, Fig. 12 that parameters a_2, k_1, a_4 have expectable behaviour – in this particular case power function dependence was recognized.

Different ways of simplification and elimination of non-important parameters were tested. In some cases satisfactory approximation could be reached by more than one way, *eg* for loops at B_a = 1 T both g_1+g_2 and g_2+g_3 combinations are successful. The latter case provides excellent approximation, at the expense that H_c has to be shifted towards H_a and it turns into one of parameters.

5 CONCLUSIONS

New analytical four-component model with flexible parameters for dynamic hysteresis loops simulation was presented. It was verified for classic electrical silicon steel at sinusoidal magnetization. The model is able to mimic the varied shapes of dynamic hysteresis loops with acceptable deviation.

The simulation quality of saturated loops seems to be weaker, but, for the most part, it is due to high slope of the loop in the H_c area. Approximation is also weaker in the region where elliptical loop evolves into sigmoid one. Attention should be paid to end-points of rounded loops where first-order derivative should be continuous.

Further work will be focused on parameters' behaviour and analytical description of their frequency and excitation dependence. Proposed model will be implemented for amorphous materials and checked out for other than sinusoidal excitation.

Besides dynamic hysteresis loops modelling, proposed model should be used especially for modelling and prediction of the magnetization characteristic H_a = f(B_a) and specific power losses p = f(B_a). It could represent a way to accelerate the measuring algorithm of KF9a.

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REFERENCES

[1] ZEMÁNEK, I. – NOVÁ, I.: Compensation Ferrometer KF9a – Universal Single Sheet/Strip and On-line Tester for Laboratory and Industrial Use, Proceedings of WMM'08, Ghent 2008, 413–429.
[2] IVÁNYI, A.: Hysteresis Models in Electromagnetic Computation, Akadémiai Kiadó, Budapest 1997.
[3] MAYERGOYZ, I. D.: Mathematical Models of Hysteresis and Their Applications: Second edition. Academic Press, New York 2003.
[4] TAKÁCS, J.: Mathematics of Hysteretic Phenomena, Wiley-VCH, Berlin 2003.
[5] WLODARSKI, Z.: Analytical Description of Magnetization Curves, Physica B **373** No. 2 (2006), 323–327.

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