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FIBER OPTIC SENSOR FOR THE SPACE DISTRIBUTION MEASUREMENT OF MAGNETIC FIELD

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The description of the model of single-mode optical fiber with nonreciprocal birefringence caused by presence of magnetic field is presented and analyzed. The model is applied to the measurement of the magnetic field distribution along the sensing single-mode optical fiber. Sensor is based on the utilisation of the polarization optical reflectometry for the mesurement of Stokes polarization parameters distribution along the fiber and the results of that measurements are futher transformed into the local birefrinegnce parametrs of the fiber and finally it allows to calculate the sought magnetic induction distribution.

Keywords: Fully distributed optical fiber sensors, Rayleigh back-scattering, Polarization Optical Fiber Relectometry, Faraday effect, Magnetic induction

1 INTRODUCTION

Nowadays Optical Time-Domain Relectometry (OTDR) that was discovered by Barnoski and Jensen in 1976 [1] and originally had been most frequently used for the measurement of space distribution of attenuation coefficient along the standard telecommunication optical fiber has undergone a strong development process. At present time there are nearly 10 various modifications of this classic „backscattering method“ in time domain or in frequency domain [2-5]. In this regard one can remind for example Coherent OTDR (C-OTDR), Low Correlation OTDR (LC-OTDR), Photon-Counting OTDR (PC-OTDR), Coherent OTDR with Synthesized Coherence Function (OTDR-SCF), Polarization OTDR (PO-OTDR), Optical Frequency-Domain Reflectometry (OFDR) and others. As compared with classic OTDR´s performance parameters (like sensitivity, space resolution, SNR) each of these specific modifications has some significant advantages but also some drawbacks and they are usually used for “suitable” specific applications where advantages prevail drawbacks. These special applications concern not only testing of optical fiber transmission systems (e.g. measurement of PMD distribution) in telecommunications but at present they are increasingly used in design and realization of so called “Fully Distributed Optical Fiber Sensors” (FDOFS). These sensors make possible to measure the space distribution of various physical quantities like mechanical stress, tension, friction, temperature, electric field, magnetic field and others that in some way can influence the local index of refraction in the fiber core along the sensing fiber. In the design of FDOFS special role plays PO-OTDR that is extremely sensitive to changes of local state of polarization (SP) of optical radiation in the fiber. As it is well known the SP can be characterized and described using the Stokes vector parameters defining the position of the point on Poincare sphere (SP) that can be precisely measured by PO OTDR. A big advantage of the SP measurement by that method is that only the intensity of optical radiation is to be really measured and not its phase that could be very difficult and perhaps in many situations quite impossible.

The main problem of PO-OTDR application in FDOFS consists in the processing of measured data defining the SP distribution along the fiber. The final goal of the sensing is to evaluate the space distribution of measured external physical quantity. To achieve this one has *to design the theoretical model* describing the mechanism of interaction of the measured external quantity with the fiber. Induced changes in refractive index distribution along the fiber are related with local changes of SP that are in defined relation with really measured SP by PO OTDR at the output of the fiber. So the characteristic measurement procedure using PO-OTDR can be described as follows: the SP of the back scattered light coming from the tested optical fiber is measured. Afterwards the measured data are processed according to the designed theoretical model that makes possible to calculate the distribution of the local birefringence of the fiber and then to define the magnitude of the measured physical quantity.

To this time various models have been suggested and applied [6-10]. The main differences between them are in taking or not into account the statistical properties of the optical fiber. It is also important to remind that all of these models assume the reciprocity of the fiber properties. It implies that all measured data using the forward propagating light wave are equivalent to those measured when using backward propagating wave. In other words there is unambiguous relation between the Jones matrix for forward (testing impulse) and backward (backscattered impulse) propagating radiation [11].

The main goal of this paper is to describe and to analyze the model of the FDOFS for the space distribution of the magnetic induction. The model is based on the application of the PO OTDR using also the PC OTDR. The main feature of the model is taking into account of Faraday rotation effect in the standard optical fiber induced by magnetic field. As it is well known [12] the efficiency of the Faraday effect in silica fibers is described by the Verdet constant V that in this case is rather small. To this time this sensor technique can be applied only for the measurement of the rather high value magnetic inductions. The main consequence of the Faraday phenomenon is that it makes the situation rather unique or as we shall see later the reciprocity has to be avoided and the non-reciprocity is to be implemented into the presented mod-

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el. In the section 2 the basic theoretical information concerning the use of Stokes formalism for the description of the relation between the birefringence and SP in homogeneous optical fiber is given. This is further developed and applied in section 3 for the formulation of the FDOFS model for the measurement of space distribution of magnetic induction using the polarization sensitive optical reflectometry.

2 BIREFRINGENCE DESCRIPTION AND ITS MEASUREMENT IN HOMOGENNEOUS STRAIGHT OPTICAL FIBER

In an ideal homogeneous isotropic single-mode optical fiber there is a possibility of propagation of two degenerated orthogonally polarized modes. It means that corresponding propagation constants of the two degenerated modes are equal and consequently all possible polarizations of radiation entering the fiber will not exchange the energy. There is no interaction between them. However in real fibers due to the index of refraction microscopic fluctuations and the influence of some external factors (pressure, tension, temperature,…) the above mentioned mode degeneration is lost. The propagation constants of the original non-interacting modes are changed and mutual coupling between modes results.

Let us define the complex amplitudes $a_x(z)$, $a_y(z)$ of both modes (plane waves) as a function of the propagation distance "z" along the z axis of the orthogonal coordinate system that is identical with the fiber axis. Then using the weak coupling approximation [14] one can write for the resulting complex amplitude of the total electric field the relation (harmonic time dependence is omitted)

$$\vec{E}(x,y,z) = a_x(z)\vec{E}_x(x,y) + a_y(z)\vec{E}_y(x,y) \qquad (1)$$

Considering the weak amplitude dependence on the z coordinate the influence of mutual coupling between modes can be described by two first order differential equations as follows.

$$\frac{d}{dz}\begin{bmatrix} a_x(z) \\ a_y(z) \end{bmatrix} = -j\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} a_x(z) \\ a_y(z) \end{bmatrix} \qquad (2)$$

where k_{ij} are mode coupling coefficients and d/dz represents the z derivative. Except of coupling coefficients k_{ij} and z the solution of these equations depends also on two initial conditions $a_x(0)$, $a_y(0)$ that define the input SP.

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = f\left(a_x(0), a_y(0), z, k_{ij}\right) \qquad (3)$$

The defined formulation is not quite suitable mainly due to the need of complex quantities measurement what is practically very difficult. Also effect of depolarization is not included. These drawbacks can be avoided by the transformation of that system of description into the system based on 4 Stokes parameters [11] defined by the above complex amplitudes as follows.

$$\begin{aligned} S_0(z) &= a_x(z)a^*_x(z) + a_y(z)a^*_y(z) \\ S_1(z) &= a_x(z)a^*_x(z) - a_y(z)a^*_y(z) \\ S_2(z) &= a_x(z)a^*_y(z) + a_y(z)a^*_x(z) \\ S_3(z) &= j\left[a_x(z)a^*_y(z) - a^*_x(z)a_y(z)\right] \end{aligned} \qquad (4)$$

Combining (4) and (2) and neglecting the optical loses ($dS_0/dz = 0$ a consequently $S_0(z) = S_0(0)$ = constant), one can write the system of differential equations for the Stokes parameters in the form:

$$\begin{bmatrix} \frac{dS_1(z)}{dz} \\ \frac{dS_2(z)}{dz} \\ \frac{dS_3(z)}{dz} \end{bmatrix} = \begin{bmatrix} 0 & 2k_2 & 2k_1 \\ -2k_2 & 0 & \Delta \\ -2k_1 & -\Delta & 0 \end{bmatrix}\begin{bmatrix} S_1(z) \\ S_2(z) \\ S_3(z) \end{bmatrix} \qquad (5)$$

where $K = k_{12} = k^*_{21} = k_1 + jk_2$ a $\Delta = k_{22} - k_{11}$.
Without depolarization it holds $S^2_0 = S^2_1 + S^2_2 + S^2_3$ and the solution of (5) makes possible to solve the problem and to get the expression describing the dependence of Stokes parameters on the distance z. If k_{ij} are supposed *not to depend on z* the system of equation (5) has the solution

$$\begin{aligned} S_0(z) &= S_0(0) = 1 \\ S_1(z) &= S_{1P}N + \left(S_1(0) - S_{1P}N\right)\cos(\delta\beta z) \pm \\ &\left(S_{3P}S_2(0) - S_{2P}S_3(0)\right)\sin\left(\delta\beta z\right) \\ S_2(z) &= S_{2P}N + \left(S_2(0) - S_{2P}N\right)\cos(\delta\beta z) \pm \\ &\left(S_{1P}S_3(0) - S_{3P}S_1(0)\right)\sin\left(\delta\beta z\right) \\ S_3(z) &= S_{3P}N + \left(S_3(0) - S_{3P}N\right)\cos(\delta\beta z) \pm \\ &\left(S_{2P}S_1(0) - S_{1P}S_2(0)\right)\sin\left(\delta\beta z\right) \end{aligned} \qquad (6)$$

where $N = S_{1P}S_1(0) + S_{2P}S_2(0) + S_{3P}S_3(0)$ and $[s_P]=[1, s_{1P}, s_{2P}, s_{3P}]$ is one of two principal state of polarization (PSP) Stokes vectors. PSP are those that do not change during the propagation. As a result in this case the state of polarization at input end is the same as at the output end of the homogeneous fiber. From (6) one can deduce that the SP vector [*S(z)*] rotates around the PSP vector [S_P] under the constant angle given by that between the [S_P] and [*S(0)*].
The PSP Stokes vector components S_{iP} can be calculated as follows:

$$S_{1P} = \pm\frac{\Delta}{\delta\beta} \quad S_{2P} = \mp\frac{2k_1}{\delta\beta} \quad S_{3P} = \pm\frac{2k_2}{\delta\beta} \qquad (7)$$

$$\delta\beta = \sqrt{\Delta^2 + 4|K|^2} \qquad (8)$$

where $\delta\beta$ is the difference of the propagation constants of the two PSP. The derived relations make possible to calculate the distribution of the local SP along the fiber or *the dependence of the SP at the output as a function of coupling coefficients that can be potentially influenced by various external physical magnitudes.* If the fiber length *z=l* is sufficiently short and the external factor is constant along that short fiber it is possible to calculate the coupling coefficients using the measured Stokes vectors [*S(z=l)*] for a given [*S(z=0)*]. If we know the mechanism of transformation of external physical quantity changes into those of coupling

coefficients it is possible to use the measured changes of corresponding Stokes vectors for the calculation of external quantity value.

It is important to stress that above achieved results are useable only if the input SP is given. Due to the fact that the PSP Stokes vector components (7) that depend on Δ and k_1 ,k_2 are generally not known it is not possible to calculate these parameters from one measurement of Stokes parameters. This problem can be solved by the introduction of the Mueller matrix formalism relating the input SP [*S(0)*] with the output one [*S(z)*] what can be ex pressed by the following linear matrix equation.

$$[S(z)] = [M(z)].[S(0)] \tag{9}$$

Where

$$[M(z)] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & S_{1P}^2 + \left(1 - S_{1P}^2\right)C & S_{1P}S_{2P}(1-C) \pm S_{3P}S & S_{1P}S_{3P}(1-C)mS_{2P}S \\ 0 & S_{1P}S_{2P}(1-C)mS_{3P}S & S_{2P}^2 + \left(1 - S_{1P}^2\right)C & S_{2P}S_{3P}(1-C) \pm S_{1P}S \\ 0 & S_{1P}S_{3P}(1-C) \pm S_{2P}S & S_{1P}S_{3P}(1-C)mS_{1P}S & S_{3P}^2 + \left(1 - S_{3P}^2\right)C \end{bmatrix} \tag{10}$$

and $C = \cos(\delta\beta z)$ and $S = \sin(\delta\beta z)$.

Matrix (10) describes the case of the most frequently appearing elliptical birefringence in optical fiber without consideration of depolarization and loses. As a result the determination of the Mueller matrix coefficients through the measurement makes possible to assign the true output SP ***S**(z)* to the arbitrary input SP vector ***S**(0)*.

3 MODEL FOR THE SPACE DISTRIBUTION MEASUREMENT OF MAGNETIC FIELD

To formulate the theoretical model of optical fiber sensor for the measurement of magnetic induction space distribution B(z) along the testing optical fiber based o the use of PO OTDR and using the effect of Faraday rotation it is suitable to rewrite the equation (5) into the following vector form

$$\begin{bmatrix} \frac{dS_1(z)}{dz} \\ \frac{dS_2(z)}{dz} \\ \frac{dS_3(z)}{dz} \end{bmatrix} = \begin{bmatrix} \beta_1(z) \\ \beta_2(z) \\ \beta_3(z) \end{bmatrix} \times \begin{bmatrix} S_1(z) \\ S_2(z) \\ S_3(z) \end{bmatrix} \tag{11}$$

where $\beta_i(z)$ are now the componenets of the *local elliptical birefringnce* vector [β(z)] of the sensig optical fiber exposed to the external magnetic field vector [B(z)] whose longitudinal projection $B_L(z)$ on the fiber axis is given as $B_L(z)=B(z).\cos\psi(z)$. Such a transcription of (5) is possible also due to the fact that Stokes vector magnitude is by definition permanently equal to 1 and therefore any its changes along *z* coordinate have to be perpendicular to the surface of the Poincare sphere. The birefringence components β_1, β_2 represent the linear birefringence which does not depend on the light propagation direction and β_3 is the circular birefringence which in reciprocal case of the fiber birefringence has opposite sign for backward direction as compared to one for forward direction. *For the non-reciprocal case as it is in presence of Faraday effect the signs for both directions are equal*. It is to be noted that equations (5),(7),(8) and (10) make possible to find the relations between the birefringence vector components and coupling coefficients and Stokes vector components that can be measured. The significant difference between the equations (5) and (11) is in the fact that (5) describes the homogeneous fiber with constant coupling coefficients along the fiber and in contrast the (11) describes the same but on an infinitesimally short section of fiber at position z with specific local coupling coefficients and PSP unique for that section. Therefore it is important to be aware that $\beta_i(z)$ are local parameters of the fiber. When using the PO OTDR method for the measurement of Stokes parameters of Rayleigh backscattered light from the small section at position *z* on the fiber it can be performed only at the input end of the fiber and consequently the measured Stokes vectors are strongly dependent on the transmission properties of the testing fiber in both directions. So in the case of nonreciprocal fiber caused by Faraday rotation effect it is necessary *to account that non-reciprocity*. Defining the forward - [$\beta_F(z)$], backward - [$\beta_B(z)$] and roundtrip birefringence - [$\beta_R(z)$] vectors that relate the forward – [$S_F(z)$], the backward – [$S_B(z)$] and round-trip – [$S_R(z)$] Stokes vectors through their corresponding forward - [$M_F(z)$], backwards - [$M_B(z)$] and round-trip [$M_R(z)$] Mueller matrices one can write for their evolution description along the fiber the similar differential equations as (11). Due to the fact that forward and backward birefringence vectors are local parameters the round-trip birefringence [$\beta_R(z)$] accumulates the birefringence influences of all fiber sections between the input and considered fiber section at distance z. Due to the fact that PO OTDR can actually measure Stokes parameters only for the round-trip dispersion we have to find the relation between the [$\beta_F(z)$], [$\beta_B(z)$] ,[$\beta_R(z)$] and [$M_F(z)$], [$M_B(z)$] , [$M_R(z)$]. Applying the equation (11) for the forward, backward and roundtrip Stokes vectors with corresponding birefringence vectors it can be found [13] that the round-trip Stokes vector evolution is described by the equation

$$\begin{aligned} \left[\frac{dS_{RT}(z)}{dz}\right] &= [\beta_{RT}(z)] \times [S_{RT}(z)] = \\ &\left[2.[M_B(z)].[\beta_F(z)]\right] \times [S_{RT}(z)] \end{aligned} \tag{12}$$

where [$M_B(z)$] is the backward Mueller matrix.

Similarly also for $[M_B(z)]$ behavior one can derive the following differential equation

$$\frac{d\left[M_B(z)\right]}{dz} = \frac{1}{2}\left[M_B\right].\left[\beta_{RT}(z)\right]\times\left[M_B(z)\right] \qquad (13)$$

The initial condition for the solution of (13) is $[M_B(0)]= [I]$, $[I]$ being the diagonal unit matrix. And finally for the round-trip birefringence vector as a function of z coordinate it is possible to find the following relation

$$\left[\beta_{RT}(z)\right] = 2.\left[M_B(z)\right].\begin{bmatrix}\beta_{1L}(z)\\ \beta_{2L}(z)\\ \beta_{3C}(z)\end{bmatrix} \qquad (14)$$

The local birefringence vector component β_{3C} that is the object of the measurement represents the non reciprocal forward circular birefringence caused by Faraday rotation. It is related to the measured magnetic field induction by the expression $\beta_{3C}(z)= 2VB(z)\cos\psi(z)$ where V is the Verdet constant, $B(z)$ is the local amplitude of the magnetic induction and $\psi(z)$ is the angle between the induction vector direction and the fiber axis at point z. From the above equation it is possible to describe the following procedure for the magnetic field distribution measurement along the sensing fiber located in the magnetic field.

1. From the equation (12) it follows - the measurement of the round-trip Stokes vector using the PO-OTDR for minimum two input Stokes vectors allows to calculate the components of the round-trip birefringence vector $[\beta_R(z)]$.
2. Having the vector $[\beta_R(z)]$ the equation (13) can be used to calculate the backward Mueller matrix $[M_B(z)]$.
3. Using the equation (14) makes possible to calculate the sought component $\beta_{3C}(z)$ because the left side of the (14) is already known.
4. And finally - application of the expression $\beta_{3C}(z)= 2VB(z)\cos\psi(z)$ provides the tool for calculation of "longitudinal" component of the sought magnetic induction.

4 CONCLUSIONS

Theoretical model for the measurement of the space distribution of magnetic induction based on the Rayleigh backscattering in nonreciprocal optical fiber exploiting the Faraday rotation phenomenon was briefly described. The special consequence of the presence of that phenomenon in sensing fiber is that due to the induced fiber non-reciprocity the third component of the total roundtrip birefringence vector that is the sum of the forward and backward contributions is not zero as it is the case of the reciprocal fiber birefringence without Faraday rotation. It was shown that there is unambiguous relation between the roundtrip circular birefringence and forward one making possible to determine the forward circular birefringence component and consequently to calculate the thought magnetic field.

However, it is important to remind that due to rather small Verdet constant V only rather strong magnetic fields (B≅1 T) are possible to measure by this approach. As for the sensitivity and space resolution at present time the optical polarization frequency domain reflectometry (PO-OFDR) is most suitable for the measurements of Stokes parameters although the Photon-Counting OTDR could be also used but the measurement time might be rather longer as compared with PO-OFDR.

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