Journal of ELECTRICAL ENGINEERING, VOL 63. NO 7s, 2012, 90-93

UNCERTAINTY ANALYSIS OF CALIBRATION OF THE 3D COIL SYSTEM

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The scalar calibration of the 3D coil system can be used as an efficient metrological method to identify all coil system's parameters. To express the method accuracy, the uncertainty of sensitivity establishing has been analytically calculated and the uncertainty of the orthogonal angles, which are numerically calculated parameters, , was evaluated with the Monte Carlo method. The method uncertainty was affected mostly by the calibration current stability and current measurements.

Keywords: uncertainty, calibration method, coil system, Monte Carlo

1 INTRODUCTION

The scalar calibration establishes six parameters (sensitivities and orthogonalities) of the 3D coil system. Only if these parameters are known, it is possible to use the coil system for the precise calibration of tri-axial magnetic sensors.

Having known the uncertainties of the mentioned parameters is important to estimate the uncertainty of the sensor calibration.

The scalar calibration published in [1] employs sequence of calibrated currents into individual coils and coil pairs plus scalar magnetometer to calculate all coil system parameters. The coil sensitivities are established by the analytical equation, while the angles between the coils (called "coil orthogonalityies") which are obtained by the non-linear optimization method.

In [1], the uncertainties of the calibrated parameters were only experimentally estimated.

The uncertainty of the coil sensitivities are derived from the standardized method according to [2] by a partial derivation of the analytical expression. Nevertheless, the more complicated case is during the uncertainty analyses of the coil orthogonalities. As the orthogonalities are calculated by the optimization algorithm, the analytical solution can not be reached and hence the analytical uncertainty analysis cannot be employed. The only way how to obtain the uncertainty is to choose the simulating method. The Monte Carlo method [3] was carried out to reveal main source of the uncertainty. The input parameters from which the angles are calculated are identified and the suitable probability distribution is selected. The values from the distribution are periodically sent to the calculated algorithm and a set of the results is recorded. The standard deviation of this set is taken as the resulting uncertainty.

2 CURRENT AND MAGNETIC FIELD MEASUREMENT

The calibration current into the coils was measured with the Agilent 34401 in the current mode and 100 PLC integration time was used to suppress AC interferences.

The Overhauser magnetometer measured the magnetic field in fixed interval of 3 seconds. That means that these two measured quantities are not synchronous.

From this reason, the current source's stability is very important because the stable current and magnetic field is assumed during the calibration. The non-stable current would add the additional uncertainty to the measured magnetic field and the magnetic field variation and current variation could not be distinguished in the measured data.

In our case, the coil is excited by the current of 1 A, coil sensitivity is about 50 nT/A and we would want to have the stability of magnetic field generated by the coil of 10 ppm. Thus the current source has to be stabilized at the level of 10 μA to reach magnetic field stability of the 0.5 nT.

3 UNCERTAINTY OF COIL SENSITIVITIES

The sensitivity is calculated from the analytical equation

$$S = \sqrt{\frac{B_{-}^{2} + B_{+}^{2} - 2B_{E}^{2}}{2I^{2}}} \tag{1}$$

The B-type standard uncertainty of the calculated value is performed by sum of the uncertainty sensitivities (partial derivation of Equation 1).

$$u_S^2 = \left(\frac{\partial S}{\partial B_{-}} u_{B-}\right)^2 + \left(\frac{\partial S}{\partial B_{+}} u_{B+}\right)^2 + \left(\frac{\partial S}{\partial B_{E}} u_{BE}\right)^2 + \left(\frac{\partial S}{\partial I} u_{I}\right)^2 \tag{2}$$

Sources of significant uncertainty in that analytical procedure were localized at all measured parameters. The current uncertainty (u_I) was established by using information from manufacture's certificates that is converted to standard deviation.

The uncertainty of the magnetic field measurement has to be specified in different way. The scalar magnetometer (Overhauser magnetometer) which we use in our method has accuracy of 0.2 nT. This value could not be taken as a

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ISSN 1335-3632

base for establishing the uncertainty because the measurements during the scalar calibration are usually carried out in non-stable Earth's magnetic field. Ideally, if the Earth's field was within the precision of the scalar magnetometer its accuracy could be converted to uncertainty of magnetic field measurement. Unfortunately, according to an obtained record of the magnetic field, depicted in the Fig. 1,

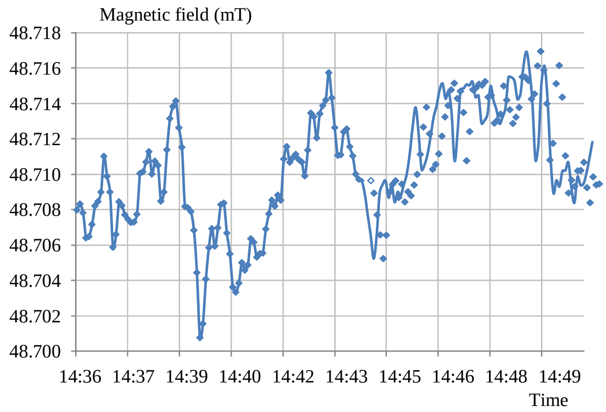

Fig. 1. Record of the magnetic field in calibration place

the basic field variations, been affected by natural influences as well as industrial noise, caused bigger uncertainty than the uncertainty of the device itself. The stability is also given by time interval, which is confined to the calibration current steps when the magnetic field is measured. We calibrated the coils in 20 seconds time steps. The magnetic field stability was established as ±5nT. This value was substituted as an input uncertainty of the magnetic field.

2.1 Analytical calculation

For analytical calculation, the particular uncertainties were established according the Table 1.

Tab. 1. Substituted uncertainty of the measured quantities

Quantity	Value	Uncertainty	Uncertainty
I (N-S coil)	0.6135 A	746/√3 µA	702 ppm
I (E-W coil)	1.0007 A	1100/√3 µA	635 ppm
I (Vertical)	0.7051 A	753/√3 µA	617 ppm
B_-, B_+, B_E	20-80 µT	5/√3 nT	144-36 ppm

If we filled all parameters into the equation 2, we calculated the B-type standard uncertainty without a coverage factor. We got the results shown in Table 2.

Tab. 2. Calculated uncertainty from analytical way, without a coverage factor

Coil	Sensitivity (nT/A)	Uncertainty (nT/A)	Uncertainty (ppm)
H (N-S)	56929	39.2	688
h (E-W)	19311	15.3	792
Z (vert)	37121	26.3	710

2.1 Monte Carlo calculation of the uncertainties

Establishing the uncertainties of the calibrated sensitivities by the analytical way is a standard process. But according to the article [3], the strong non-linearity of equation from which the results is calculated can cause that the sensitivity analysis of the uncertainty do not approximated exactly its propagation through the equation.

Therefore we decided also to test the uncertainty analysis with the Monte Carlo simulation. The Monte Carlo analysis is a numerical method for combining the input quantities distribution and their propagation through the equation. The advantage is that the resulted distribution can be depicted and so the uncertainty can be estimated more precisely.

The input parameters to Monte Carlo method are probability distribution of the quantities from which the result is calculated. The only input quantity limits are known from devices errors and thus the uniform probability distribution was selected. The measured value was as a mean value and their limits are bound of the uniform (rectangular) distribution.

The quantities were randomly generated from these distributions and periodically substituted into the equation (1). This procedure was performed many times to get the suitable preview about the distribution of calculated parameter. The resulted simulated uncertainties are shown in Table 3. The uncertainty was calculated as a standard deviation of the sets of the results.

Tab. 3. Simulated uncertainty of the sensitivities by Monte Carlo method

Coil	Sensitivity (nT/A)	Uncertainty (nT/A)	Uncertainty (ppm)
H (N-S)	56929	39.1	687
h (E-W)	19311	15.3	792
Z (vert)	37121	26.4	712

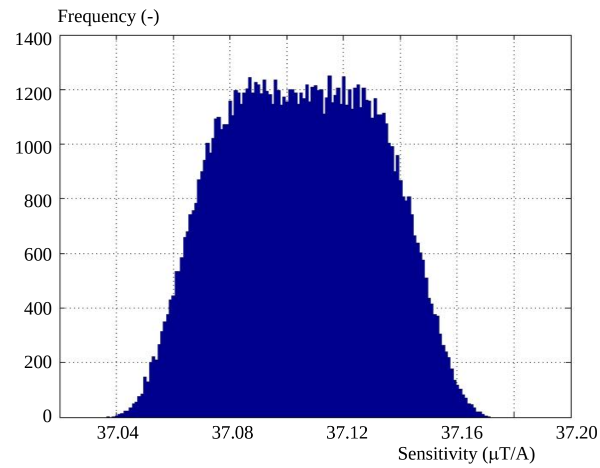

Fig. 2. Histogram of simulated results for coil Z (10000 samples).

We can conclude, that the results of analytical calculation were fully confirmed by simulations.

5 UNCERTAINTY OF ORTHOGONALITIES

The angular imperfection of the coil system can be also established by the scalar method. In comparison with establishing coil sensitivities, the orthogonalities cannot be calculated analytically, because they are established by the optimization method.

Therefore, the classical analytical approach to uncertainty analysis could not be used because the sensitivity to the individual error sources cannot be derived.

One choice is not to express the B-type uncertainty and exploit only A-type by using the multiple measurements but thus we would not have knowledge of the main source of uncertainties.

We therefore here also used the Monte Carlo analysis. The qualification and quantification steps were carried out and into the input source of uncertainty was added the uncertainty of the sensitivity which was established in the previous step.

Tab. 4. Simulated uncertainty of orthogonalities by Monte Carlo method

Coils	Angle (deg)	Uncertainty (deg)	Uncertainty (ppm)
H-h	91.15	0.055	598
Z-H	89.95	0.089	988
Z-h	89.97	0.088	987

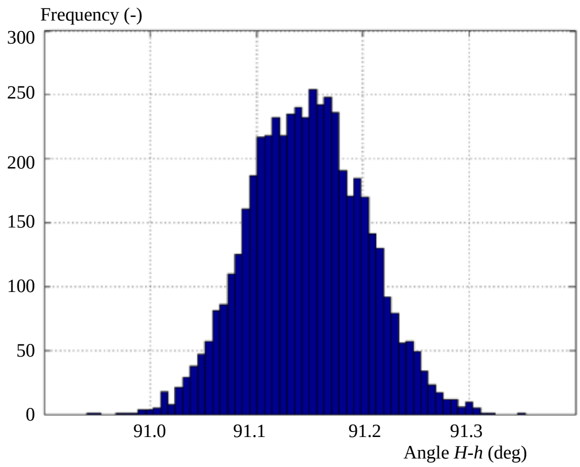

Fig. 3. Histogram of simulated results angle between *H-h* coil (5000 samples)

5 DISCUSION

In establishing the coil sensitivities, a main source of uncertainty is the current measurement. Even if the magnetic field measurements has only about times better uncertainty, the current has main influence because according to the uncertainty analytical analysis Therefore, small changes of the current can cause course uncertainty of calculated coil sensitivity.

This influence was also proved with the Monte Carlo method.

The uncertainty of the angular establishing is mainly affected by the current accuracy but also previous established sensitivity. These facts point to improve the current measurement.

5 CONCLUSIONS

In the setup, which the coil system was calibrated with, the uncertainty of establishing the sensitivities was about 700 ppm and then the corresponding uncertainty of orthogonalities was determined by the simulation in range of 600-1000 ppm. These values could be sufficient for some application.

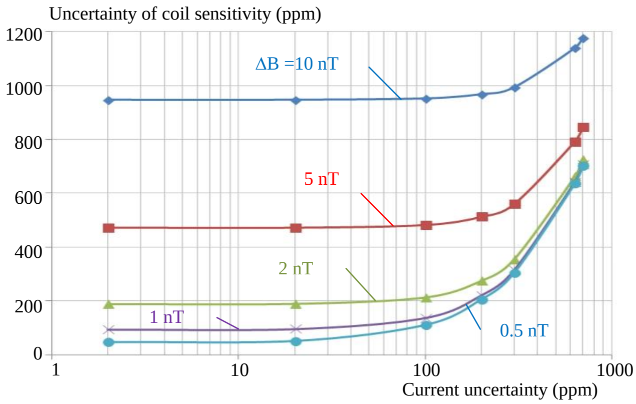

Fig. 4. Dependency of the uncertainty of the coil sensitivity on the input uncertainties (for the Z coil)

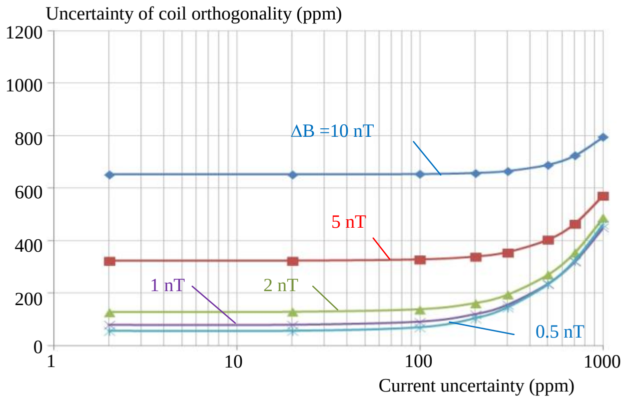

Fig. 5. Dependency of the uncertainty of the coil orthogonality on the input uncertainties, the sensitivity uncertainty is constant 100 ppm

To improve the accuracy of this method, the current measurement was upgraded.

If the ammeter was replaced by a very precise shunt resistor (with 50 ppm uncertainty) and the current was measured as a voltage drop (with 20 ppm uncertainty) across the resistor, we would be able to determine the current more accurate (with 40 ppm) and also the sensitivity would have lower uncertainty of 473 ppm. For the Z coil sensitivity's uncertainty, the dependency is shown in Figure 2.

At the certain point, the current uncertainty starts not influencing the resulted uncertainty because the uncertainty of the magnetic field dominates. Therefore, it would be also reasonable to improve the magnetic field stability for example by the compensation loop. The lower uncertainty of the current and sensitivity will be reflected in the orthogonality uncertainty.

By observing the dependency of orthogonality uncertainty another input uncertainty is added, the uncertainty of sensitivity. Therefore, two dependencies are necessary to reveal main influences of the angle accuracy. Figures 5 and 6 show the dependencies. If the sensitivity uncertainty is constant of 100 ppm, within the same magnetic field error the resulted uncertainty is not decreased from the 100 ppm current uncertainty.

Both dependencies depict that to minimize the uncertainty of the sensitivity, the input uncertainty should be at the same level.

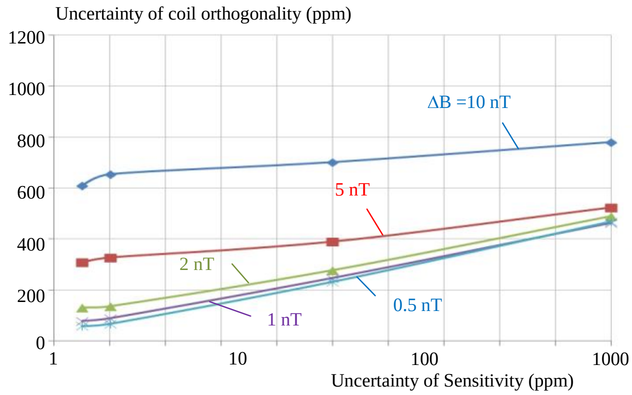

Fig. 6. Dependency of the uncertainty of the coil orthogonality on the input uncertainties, the current uncertainty is constant 100 ppm

Acknowledgement

The work was supported by the project METMAGS of the European Metrology Research Programme, by the research grant No. SGS12/194/OHK3/3T/13 sponsored by Czech Technical University in Prague and also by the grant of the technology Agency of Czech Republic No. TA01010298.

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Received 24 August 2012

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