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NEW THERMAL LINK FINITE ELEMENT FOR FGM CONDUCTORS

Juraj Hrabovský* — Juraj Paulech* — Justín Murín*
Vladimír Kutiš* — František Janíček*

A new effective thermal link finite element is presented in this paper, which is suitable for thermal analysis of the conductors made of a functionally graded material (FGM) with spatial continuous variation of material properties, continuously distributed generated heat along the element length and continuous variation of the convection from the element surface. Numerical experiment will be done concerning calculation of the thermal fields of the FGM conductor. The solution results will be discussed and compared with those obtained using a very fine mesh of the 3D solid finite elements of the FEM program ANSYS.

Keywords: functionally graded material, thermal analysis, heat convection, FGM link finite element, ANSYS, Mathematica

1 INTRODUCTION

This contribution deals with derivation of a new link finite element for thermal analysis including heat generation in the link and heat convection from the element surface. This derivation of the thermal link element is based on exact solution of differential equation for heat transfer. The solution uses new approach in the derivation of FEM equations for link element and also includes heat convection that was not used in our previous works [1, 2]. Numerical experiment will be done in order to compare calculated results with those obtained by simulation using classic FEM elements in program ANSYS.

2 THEORETICAL BACKGROUND FOR HEAT TRANSFER

Let us consider a two nodal straight thermal link element with length L, cross-sectional area $A(x)$ and thermal conductivity $\lambda(x)$. We also consider heat generation $Q(x)$ in the link volume and heat convection $\alpha(x)$ from the link surface. Ambient temperature is T_{ok} and general boundary conditions are denoted as *OP1* and *OP2* (Fig. 1).

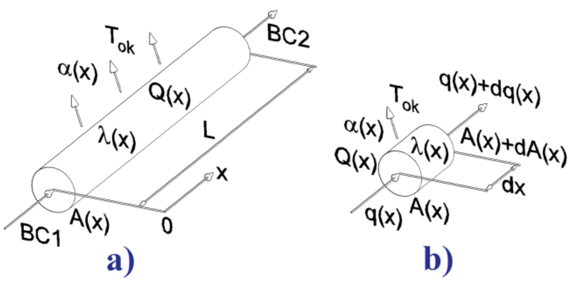

Fig. 1. Thermal link element

According to [3] we can write differential equation for heat transfer

$$-\lambda(x)\frac{d^2T(x)}{dx^2}-\frac{d\lambda(x)}{dx}\frac{dT(x)}{dx}+\alpha(x)T(x)\frac{o}{A}=Q(x)+\alpha(x)T_{ok}\frac{o}{A} \quad (1)$$

where o is the perimeter of the link.

3 FEM EQUATIONS DERIVATION

Method for solving differential equations with arbitrary order, inconstant coefficients and right side is described in [4]. The method is performed by computer code that does not use high level mathematical operations (like numeric derivation or integration). This method is suitable only for 1D problem.

Let our differential equation has this form

$$\sum_{u=0}^{m}\eta_u(x)y^{(u)}(x)=\sum_{j=0}^{g}q_j a_j(x) \quad (2)$$

where, considering also (1), we can write:

$m=2$	order of the differential equation
$y(x)\equiv T(x)$	unknown function of x is the temperature function
$y^{(u)}(x)\equiv\frac{d^uT(x)}{dx^u}$	u^{th} derivation of unknown function
$\eta_0(x)=\alpha(x)\frac{o}{A}$	zero derivation of temperature contains heat convection, see (1)
$\eta_1(x)=-\frac{d\lambda(x)}{dx}$	inconstant coefficient in the 1^{st} derivation
$\eta_2(x)=-\lambda(x)$	inconstant coefficient in the 2^{nd} derivation
$g>0$	order of right side of the dif. equation
q_j	constant coefficient in the j^{th} power of polynomial right side
$a_j(x)=\frac{x^j}{j!}$	auxiliary function for polynomial formulation

Variable x goes from 0 to length L. Inconstant coefficients $\eta_u(x)$ on the left side and right side itself have to be in polynomial form.

According to [4], the solution of (2) is in the form

$$y^{(u)}(x)=\sum_{j=0}^{m-1}y_0^{(j)}c_j^{(u)}(x)+\sum_{j=0}^{g}q_j b_{j+m}^{(u)}(x) \quad (3)$$

where order of derivative u goes from 0 to m. The program automatically calculates all transfer coefficients $c_j^{(u)}(x)$ and $b_{j+m}^{(u)}(x)$ for chosen point x.

As we can see from (1), thermal conductivity, heat generated and convective heat transfer coefficient are functions of variable x. Typical applications where such behavior of

*Slovak University of Technology, Faculty of Electrical Engineering and Information Technology, Institute of Power and Applied Electrical Engineering, Ilkovičova 3, 812 19 Bratislava1, Slovakia, juraj.hrabovsky@stuba.sk, juraj.paulech@stuba.sk, vladimir.kutis@stuba.sk, justin.murin@stuba.sk, frantisek.janicek@stuba.sk

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material properties can be observes are MEMS applications with Functionally Graded Material (FGM). Material properties of link element are changing in longitudinal (x) direction - see Fig. 1, according to chosen function. This functional grading can be also considered in lateral direction but material variation in this direction has to be transformed into 1D system by homogenization processes [1, 2].

Solution (3) for our case is

$$T(x) = \sum_{j=0}^{1} T_0^{(j)} c_j(x) + \sum_{j=0}^{g} \varepsilon_j b_{j+2}(x) = \\ = c_0(x)T_0 + c_1(x)T_0' + \sum_{j=0}^{g} \varepsilon_j b_{j+2}(x) \quad (4)$$

where $T_0 = T(x)|_{x=0}$ and analogically for derivative T_0'.

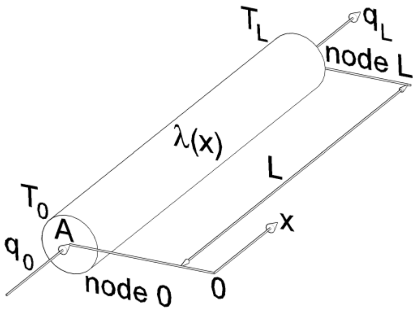

Fig. 2. Two-node link element

Let us consider two-node link element according to Fig. 2. In the first step of FEM equations derivation let us consider two boundary conditions $T(L) = T_L$ and $q(0) = q_0$. Fourier constitutive law for heat transfer for 1D problem is in the form

$$q(x) = -\lambda(x)T'(x) \quad (5)$$

Applying boundary conditions and equation (5) into (4) and after some mathematical operations we get matrix form for the 1st FEM equation

$$\begin{bmatrix} c_0(L) & -1 \end{bmatrix} \begin{bmatrix} T_0 \\ T_L \end{bmatrix} = \left[\frac{c_1(L)}{\lambda_0} q_0 - \sum_{j=0}^{g} \varepsilon_j b_{j+2}(L) \right] \quad (6)$$

Now, let us consider $T(0) = T_0$ and $q(L) = q_L$. Applying these boundary conditions into (4) with help of (3) and (5) we get 2nd FEM equation. Then we can write both these equations in the system of FEM equations in the form

$$\begin{bmatrix} c_0(L) & -1 \\ -\left(c_0(L) - \frac{c_1(L)c_0'(L)}{c_1'(L)} \right) & 1 \end{bmatrix} \begin{bmatrix} T_0 \\ T_L \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{c_1(L)}{\lambda_0} q_0 - \sum_{j=0}^{g} \varepsilon_j b_{j+2}(L) \\ \frac{c_1(L)}{c_1'(L)\lambda_L} q_L - \frac{c_1(L)}{c_1'(L)} \sum_{j=0}^{g} \varepsilon_j b_{j+2}'(L) + \sum_{j=0}^{g} \varepsilon_j b_{j+2}(L) \end{bmatrix} \quad (7)$$

The system (7) is suitable for calculation of nodal values of T and q. Next, temperature in the position x can be expressed by following equation

$$T(x) = \left[c_0(x) - c_0(L)\frac{c_1(x)}{c_1(L)} \right] T_0 + \left[\frac{c_1(x)}{c_1(L)} \right] T_L - \\ - \frac{c_1(x)}{c_1(L)} \sum_{j=0}^{g} \varepsilon_j b_{j+2}(L) + \sum_{j=0}^{g} \varepsilon_j b_{j+2}(x) \quad (8)$$

When the primary variable (temperature) is known, the secondary variables (eg. heat flux) can be calculated. According to [1, 2] we can express heat flux for homogenized link element $q^H(x)$. But more accurate is to determine this heat flux for the individual layers $q_k(x)$

$$q_k(x) = \frac{\lambda_k^L(x)}{\lambda^H(x)} q^H(x). \quad (9)$$

4 NUMERICAL EXPERIMENT

The thermal FGM conductor has been considered, see Fig. 3. Their square cross-section is constant with height $h = b = 6.66$ mm and length $L = 100$ mm.

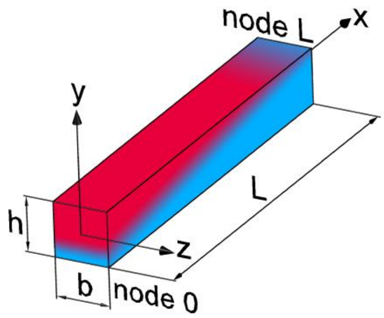

Fig. 3. FGM conductor

The FGM of this link is made by mixing two components. Material properties of the components are assumed to be constant and their values are: matrix (index m) heat conduction $\lambda_m(x,y) = 1.33$ [$\mathrm{Wm^{-1}K^{-1}}$] and fibre (index f) $\lambda_f(x,y) = 450$ [$\mathrm{Wm^{-1}K^{-1}}$]. Variation of the fibres volume fraction has been chosen as the polynomial function of x, y:

$$v_f(x,y) = 0.692 - 136.786x^2 + 911.905x^3 + 145y - 6.407.10^4x^2y + 4.271.10^6x^3y - 3\,750y^2 + 4.789.10^6x^2y^2 - 3.193.10^6x^3y^2 - 3{,}6.10^6y^3 + 1{,}08.10^9x^2y^3 - 7{,}2.10^9x^3y^3$$

This change of fibre volume fraction is shown in Fig. 4. There are also shown 11 functions for chosen layers used during homogenization process.

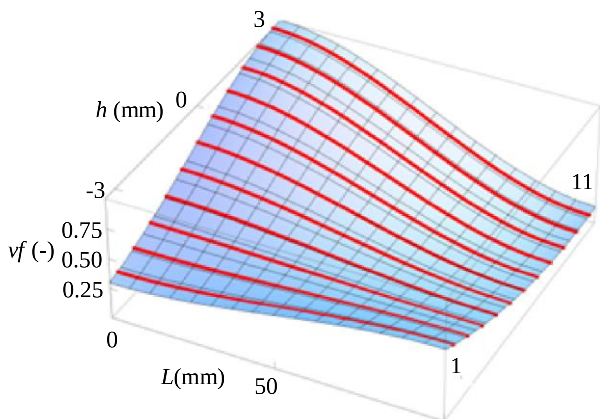

Fig. 4. Spatial 2D fibre volume fraction change

The thermal conductivity of the homogenized link has been calculated by expressions derived in [1, 2], and for $\lambda^H(x)$ we can write

$$\lambda^H(x) = \frac{2\,217\,793}{7\,260} - \frac{6\,470\,895}{121}x^2 + \frac{43\,139\,300}{121}x^3 \quad [\mathrm{Wm^{-1}K^{-1}}]$$

Boundary conditions and loads are

$t(0) = 10$ °C

$$q(L) = -1{,}56.10^4 \text{ Wm}^{-2}$$
$$Q(x) = 10^4 + 6{,}5.10^7 x - 6{,}3.10^9 x^3 \text{ Wm}^{-3}$$
$$\alpha(x) = 100 + 446\,250x - 1{,}8275.10^7 x^2 + 2{,}44375.10^8 x^3 - 1{,}0625.10^9 x^4 \text{ Wm}^{-2}\text{K}^{-1}$$
$$t_{ok} = 10\text{ °C}$$

They are also shown in Fig. 5.

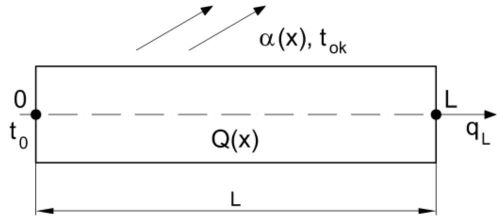

Fig. 5. Boundary conditions and loads

Form above derived equations (7-8) for our new link element, we calculated temperature for nodal values and chosen points in the field of the link element in 39 calculation points. Only one our new two-nodal finite element was used for calculation of this problem.
Because the example is relatively simple, we can use for verification purposes not only commercial FEM software ANSYS [5] but also software Mathematica [6]. In the code ANSYS the same problem has been solved using a very fine mesh – 100 000 of 3D SOLID90 elements - see Fig. 6.

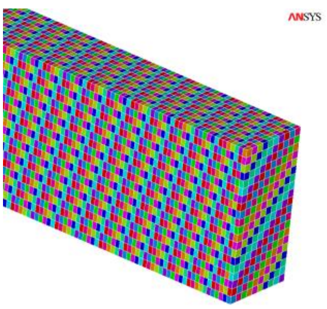

Fig. 6. Mesh of 3D elements in ANSYS

In the code Mathematica, 1D equation of heat transfer was directly solved by numerical solver of differential equations. The obtained results for temperature and heat fluxes (only layers No. 1, 6, 11 are presented) of our new approach and both verification models are shown in Fig. 7, Fig. 8 and Tab. 1. In Tab. 1 only nodal results are shown.

Tab.1. Calculated values for nodal points

	t_L [°C]	$q_{1,0}$ [Wm^{-2}]	$q_{6,0}$ [Wm^{-2}]	$q_{11,0}$ [Wm^{-2}]	q_0^H [Wm^{-2}]
New element	12.689	-6 299.1	-13 631.7	-19 612.9	-13 361.4
ANSYS	12.672	-6 494.8	-13 875.7	-19 777.6	N/A
Mathematica	12.689	N/A	N/A	N/A	-13 361.4

As we can see from obtained results, our new approach reached the same results as those calculated by code Mathematica. When we compare our results with ANSYS 3D element results, we can see small differences specially at the right end of link, that are caused by 3D effects modelled in code ANSYS. These effects cannot be naturally included in 1D model.

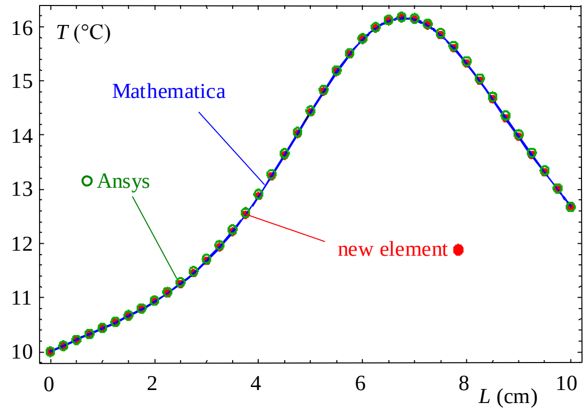

Fig. 7. Temperature in the link element

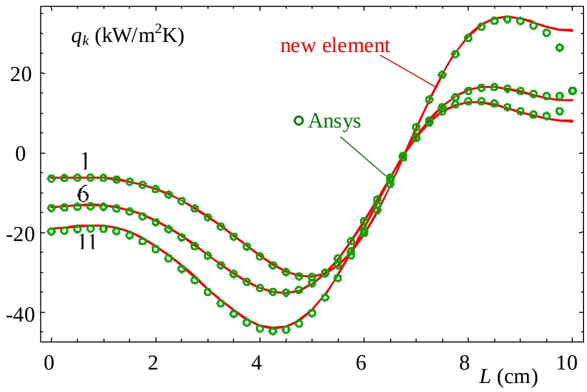

Fig. 8. Heat fluxes in the link element layers

5 CONCLUSIONS

The paper presents new approach for solution of thermal field in FGM conductors. From the results of the numerical experiment we can see an excellent agreement between the only one new 2-nodal thermal link element solution and both verification models, that were created in codes ANSYS and Mathematica. The new element solution is accurate 1D solution of the differential equation of heat transfer conduction including heat generation and convective heat transfer. As was shown in the paper. the efficiency and accuracy of this new finite element is excellent.

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Juraj Hrabovský (Ing) graduated from the Faculty of Civil Engineering, Slovak University of Technology, in 2010 from Mathematical and Computational Modeling. He is now PhD student of this faculty.

Juraj Paulech (Ing) graduated from the Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in 2009. He is now PhD student of this faculty.

Justín Murín (Prof, Ing, DrSc) was born in Oravská Lesná in 1951. He graduated from the Slovak University of Technology, Faculty of Mechanical Engineering, in 1975. He received PhD degree and DrSc degree in Applied mechanics in 1979 and 1994, respectively. In 1987 and 2001 he was appointed Associate Professor and Full Professor in Applied Mechanics. His teaching and research activities include analytical and numerical methods in mechanics and electro- thermo-mechanics. Since 1985 he is employed at the Faculty of Electrical Engineering and Information Technology STU. From 1994 to 1998 he was employed at the TU Vienna. At present he is the head of the Department of Applied Mechanics and Mechatronics at the FEI STU and subhead of the Institute of Power and Applied Electrical Engineering of FEI STU. He is cochairman of the Main Board in the Slovak Society for Mechanics. He is also the chair of Slovak branch of the Central European Association for Computational Mechanics (CEACM).

Vladimír Kutiš (doc, Ing, PhD) graduated from the Slovak University of Technology, Faculty of Mechanical Engineering in 1998. He obtained the PhD degree in applied mechanics from the same University, Faculty of Electrical Engineering and Information Technology in 2002. Since 2002 he is with the Faculty of Electrical Engineering and Information Technology of the Slovak University of Technology holding now position of associate professor. His main research interests include numerical methods in mechanics and thermomechanics as well as coupled field analysis.

František Janíček (Prof, Ing, PhD) graduated from the Faculty of Electrical Engineering (EF) of the Slovak University of Technology (SVŠT, later STU) in 1979; in the same year he took the position of Assistant Lecturer at the Department of Power Engineering of EF SVŠT). In 1984, he obtained a PhD in the field of Power Engineering at EF SVŠT. In 1989, he was appointed as a senior lecturer and on 30 September 1999, he received the academic degree of Professor. He held the Office of the Dean of the Faculty of Electrical Engineering and Information Technology of STU in Bratislava from 1 February 2007 till 31 January 2007. He has been the Vice-Rector for Development of the Slovak University of Technology since 1 February 2007. He is a leading expert in the area of power engineering. He has authored or co-authored thirteen university textbooks and lecture books, four specialist monographs, more than 50 articles in journals and more than 100 presentations at international scientific conferences (of which 26 were guest lectures). He has put over 80 technical projects into practice. He is one of the co-founders of the internet-based information and education project INFELEN (information on power engineering), the founder of the EE journal where he also acts as a member of the editorial board. He is a member of the Slovak WEC SR committee, a statutory representative of the EE Association (8 organizations) and a member of scientific boards of several universities in the SR. He reviewed several major projects in the field of liberalized power market and was a member of a team appointed to develop the Energy Policy of the SR. Since 1 February 2006 he has been in the position of Chairman of Board for Technical Science with the Slovak Research and Development Agency.