Journal of ELECTRICAL ENGINEERING, VOL 63. NO 7s, 2012, 164-167

MODELLING OF THE FREE VIBRATION OF THE COMPOSITE MICRO-ELECTRO-MECHANICAL SYSTEMS

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The micro-electro-mechanical devices, like the sensors and actuators, are very often used in the automotive electronics and mechatronics. The body of the devices is built as the single micro-beam, -plate and -shell as the micro-mechanical structures. The knowledge of the dynamic properties, like the eigenfrequency and eigenmodes, is very important from responsibility and safe functionality point of view. The results of modal analysis of chosen beam micro-actuator made of the composite material with varying material properties are presented in the proposed contribution. Effect of electro-thermo-mechanical axial force on the actuator eigenfrequency has been studied and evaluated. Our new FGM beam finite element is used in the proposed modal analysis. Obtained results have been verified by 3D solid finite element.

Keywords: micro-electro-mechanical actuator, functionally graded material, modal analysis, FGM beam finite element

1 INTRODUCTION

Mechanical resonance of mechatronic devices is in most cases a negative phenomenon. If the frequency of the harmonic loads (mechanical, electrical-thermal or electromagnetic) coincide with the device eigenfrequency, the amplitude of mechanical vibration rises theoretically into infinity, which can lead to device inoperability or destruction. Therefore, the calculation of the device eigenfrequencies is straightforward. If the device is made of homogeneous material, the classical finite elements can be used in modal analysis. The multilayer beams or the beams with spatial continuous variation of material properties (functionally graded materials - FGMs) have been used in the micro-electro-mechanical systems (MEMS) design in the last time [1]. Modelling of such structure by classical finite elements is very difficult in such case, because each finite element has to be of other material properties. The finite element mesh has to be very fine, which makes the preparing of the task input data very time consuming. New approaches, mostly semi analytical were developed for analysis of such FGM single beams [2, 3]. A large disadvantage of these approaches is that the analysis of devices built as a beam structure is not available.

Therefore, in contribution [4], new finite element was developed which is able for modal analysis of MEMSbuilt as a plane beam structure. Our new beam finite element will be briefly described in the next chapter, and will be again used in modal analysis of chosen FGM actuator.

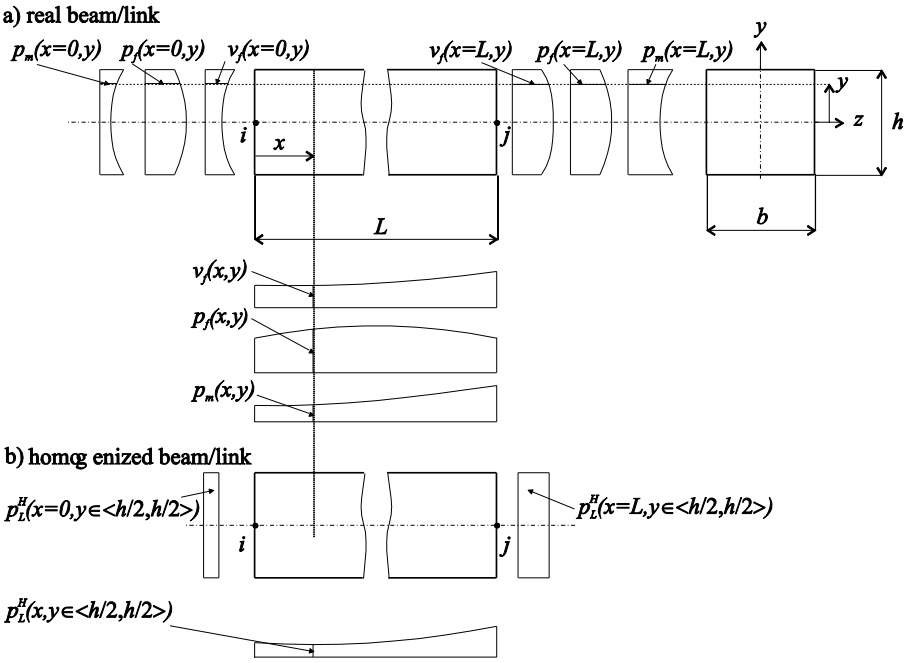

Fig. 1. Real and homogenized FGM beam/link element

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2 THE FGM BEAM FINITE ELEMENT EQUATION

Let us consider a two nodal straight beam element with constant rectangular cross-sectional area $A = bh$ and quadratic moment of inertia $I = bh^3/12$, see Fig.1.

The functionally graded material of this beam arises from mixing two components (formally named as matrix and fibres) that are approximately of the same geometrical form and dimensions.

The continuous spatial variation of the effective material properties can be caused by continuous spatial variation of both the volume fraction and material properties of the FGM constituents, or by inhomogeneous temperature field. Both the fibers volume fraction $v_f(x,y)$ and the matrix volume fraction $v_m(x,y)$ are chosen as polynomial functions of *x*, and with continuous and symmetrical variation through its height *h* with respect to the neutral plane of the beam.

The volume fractions are assumed to be constant through the cross-section depth *b*. At each point of the beam it holds: $v_f(x,y)+v_m(x,y)=1$. The values of the volume fractions at the nodal points are denoted by indices *i* and *j*.

The material properties of the constituents (fibres - $p_f(x,y)$ and matrix - $p_m(x,y)$) vary analogically (depending on inhomogeneous temperature field for example) as stated by the variation of the volume fractions. For effective material property $p(x,y)$ in the real beam we have got

$$p(x,y)=v_f(x,y)p_f(x,y)+v_m(x,y)p_m(x,y). \qquad (1)$$

The material properties are isotropic in each material point, but they vary as a function of the material point coordinates.

In our case the elasticity modulus $E(x,y)$, Poisson ratio $\nu(x,y)$, and mass density $\rho(x,y)$ have been calculated by expression (1).

The FGM shear modulus can be calculated by expression:

$$G(x,y)=\frac{E(x,y)}{2(1+\nu(x,y))}. \qquad (2)$$

If the constituents Poisson's ratio are approximately of the same value and the constituents volume fraction variation is not much strong, then the FGM shear modulus can by calculated using a simplification

$$G(x,y)=\frac{E(x,y)}{\xi}, \qquad (3)$$

where ξ is an average value of the function

$$\xi(x,y)=2(1+\nu(x,y)),$$

$$\xi=\frac{1}{L}\int_0^L\left(\frac{1}{h}\int_{-h/2}^{h/2}\xi(x,y)dy\right)dx. \qquad (4)$$

In the homogenization of the spatial varying material properties (1) the direct integration method will be used [5]. From the assumption that the respective property (e.g. stiffness) of the real beam must be equal to the analogical property of the homogenized beam, the homogenized longitudinal elasticity modules for: tension – compression $E_L^{NH}(x)$, bending $E_L^{MH}(x)$, shear $G_L^H(x)$, and the homogenized mass density $\rho_L^H(x)$ can be calculated, respectively [4]. The homogenized beam finite element including the internal variables, geometric and material properties, inertial loads is shown in the Figure 2.

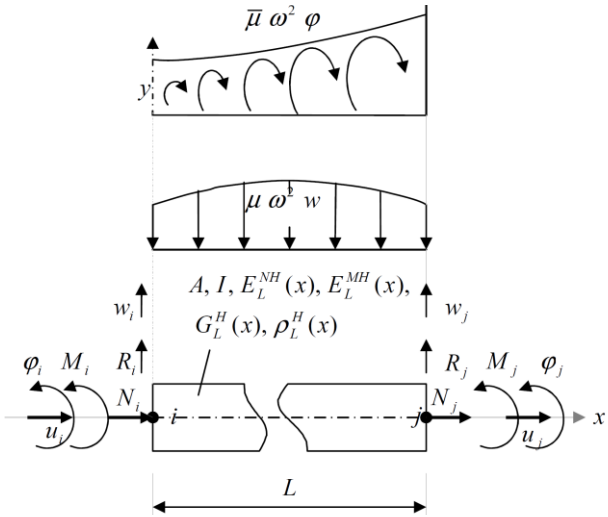

Fig. 2. Homogenized beam finite element (internal variables, geometry and material properties, inertial loads)

Here, μ is the mass distribution, $\bar{\mu}$ is the mass inertial moment distribution, ω is the natural frequency, *N* is the axial force, *R* is the transversal force, *M* is the bending moment, *u* is the axial displacement, *w* is the transversal displacement, and φ is the angle of cross-section rotation.

Derivation of the local finite element equation (4) is described in [4] in detail.

$$\underbrace{\begin{bmatrix} N_i \\ R_i \\ M_i \\ N_j \\ R_j \\ M_j \end{bmatrix}}_{\mathbf{F}^{loc}} = \underbrace{\begin{bmatrix} B_{1,1} & 0 & 0 & B_{1,4} & 0 & 0 \\ 0 & B_{2,2} & B_{2,3} & 0 & B_{2,5} & B_{2,6} \\ 0 & B_{3,2} & B_{3,3} & 0 & B_{3,5} & B_{3,6} \\ B_{4,1} & 0 & 0 & B_{4,4} & 0 & 0 \\ 0 & B_{5,2} & B_{5,3} & 0 & B_{5,5} & B_{5,6} \\ 0 & B_{6,2} & B_{6,3} & 0 & B_{6,5} & B_{6,6} \end{bmatrix}}_{\mathbf{B}^{loc}} \cdot \underbrace{\begin{bmatrix} u_i \\ w_i \\ \varphi_i \\ u_j \\ w_j \\ \varphi_j \end{bmatrix}}_{\mathbf{U}^{loc}}. \qquad (5)$$

The non-constant parameters $B_{i,j}$ (functions of natural frequency ω, the 2nd order beam theory axial force N^{II} and other beam parameters) of the local finite element matrix $\mathbf{B}^{loc}$ are not expressed in detail here from space spending point of view. $\mathbf{F}^{loc}$ and $\mathbf{U}^{loc}$ is the vector of the local forces and vector of the local displacements, respectively. When local matrix $\mathbf{B}^{loc}$ is known the global finite element matrix $\mathbf{B}^{glob}$ (according the global coordinate system) can be derived

The global matrix $\mathbf{B}^{glob}$ is obtained by usual transformation of the local matrix $\mathbf{B}^{loc}$. Transformation matrix $\mathbf{T}$ has a form

$$\mathbf{T}=\begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6)$$

For global coordinate system we get

$$\mathbf{F}^{glob}=\mathbf{B}^{glob}\mathbf{U}^{glob}. \quad (7)$$

Here $\mathbf{F}^{glob}$ and $\mathbf{U}^{glob}$ is the vector of the global forces and vector of the global displacements, respectively.

The natural eigenfrequency ω_{Ki} has been iterative calculated by software MATEMATICA [6]. The natural eigenfrequency ω will be increased until all the boundary conditions have been fulfilled. In this state, the natural frequency ω responds to the natural eigenfrequency ω_{Ki}. As the natural eigenfrequency is known, the eigenfrequency can be calculated by a usually way.

3 NUMERICAL EXPERIMENT

The electro-thermo-mechanical FGM microactuator has been considered (as shown in Fig. 3). The local coordinate system of the beams is denoted by *x*, *y*. The global coordinate system is denoted by *X*, *Y*. The actuator consists of two parts – Beam1 and Beam2. Their square cross-section is constant with height $h=10\,\mu\text{m}$ and width $b=10\,\mu\text{m}$. Lengths of the parts are: $L_1=L_2=600\,\mu\text{m}$.

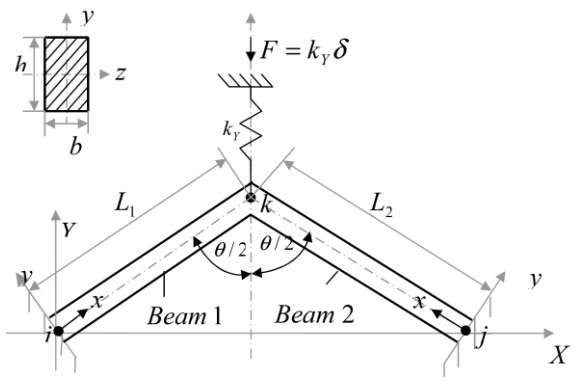

Fig. 3. FGM MEMS microactuator

The Joule's heat produces the reaction force *F* which depends of the spring stiffness k_Y. The vertical displacement of the point *k* is denoted by δ. The reaction force *F* induces the internal axial force *N* which influences the actuator stiffness and the eigenfrequency of the actuator.

The angle between the beams is $\theta=120°$. Material of the beams consists of two components: aluminum Al6061-TO as a matrix and titanium carbide TiC as a fibre.

Material properties of the components are assumed to be constant and their values are: aluminum Al6061-TO (matrix) – the elasticity modulus $E_m=69.0\,\text{GPa}$, the mass density $\rho_m=2700\,\text{kgm}^{-3}$, the Poisson's ratio $\nu_m=0.33$; titanium carbide TiC (fibres) – the elasticity modulus $E_f=480.0\,\text{GPa}$, the mass density $\rho_f=4920\,\text{kgm}^{-3}$, the Poisson's ratio $\nu_f=0.20$ [7]. The 2nd order beam theory ($N^{II}\neq 0$, the tensional axial force increases and the compression force decreases the beam stiffness) has been taken into account.

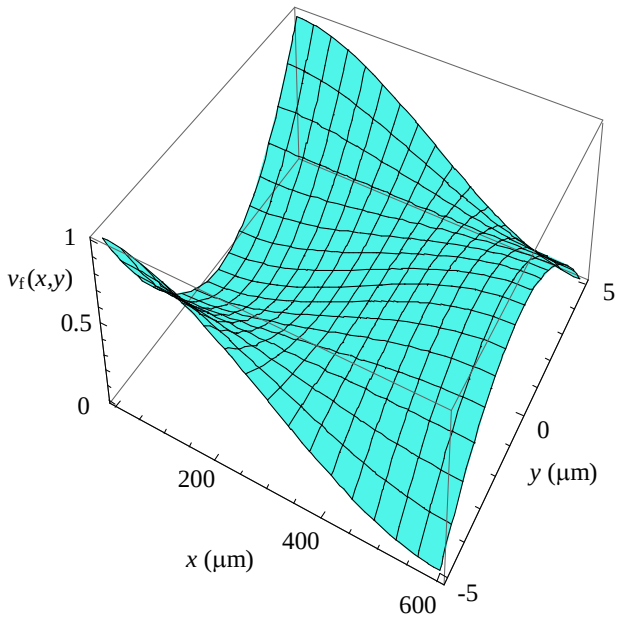

Fig. 4. Volume fraction variation - planar variation along the beam length and height.

Variation of the fibres volume fraction has been chosen as the polynomial function of the local coordinate system *x*, *y*

Beam 1 and 2:

$$v_f(x,y)=\frac{5}{900000}x^2-\frac{1}{162000000}x^3+0{,}04y^2 -\frac{5}{9000000}x^2y^2+\frac{1}{1620000000}x^3y^2$$

that is drawn in Fig.4. The same variation of the fibres volume fraction at the point *k* has been assumed.

The effective material properties of the homogenized beam have been calculated [4] and we have got:

Beam 1 (the local axis *x* begins at node *i* and ends at point *k*, Beam 2 (the local axis *x* begins at node *j* and ends at point *k*) – see Figure 3:

$$E_L^{NH}(x)=-4.228395061728403\times10^{17}x^3 +3.805555555\times10^{14}x^2+2.06\times10^8 \quad [\text{kPa}]$$

$$E_L^{MH}(x)=1.268518518518519\times10^{18}x^3 -1.141666666\times10^{15}x^2+3.156\times10^8 \quad [\text{kPa}]$$

$$G_L^H(x) = 8.15909138\times10^7 + 1.3921341942\times10^9 x + 8.853705245179\times10^{13}x^2 + 3.0506032718188\times10^{16}x^3 - 1.64321492809775\times10^{20}x^4 \quad [\text{kPa}]$$

$$\rho_L^{MH}(x) = -2.283950617283951\times10^{12}x^3 + 2.055555555\times10^9 x^2 + 3440.0 \quad [\text{kgm}^{-3}]$$

The average shear correction factor [4] for both beams is k^{sm} = 0.75. The FGM microactuator (Figure 3) - clamped at the nodes *i* and *j* has been studied by modal analysis. Effect of the varying axial force on the actuator eigenfrequency has been evaluated. The maximal axial force has been chosen to be smaller than the critical buckling force that is $N_{Ki}^{II} = -0.0108576$ N. To show the effect of axial force *N* on the eigenfrequency, its positive value (tension), negative value (compression) and equal to zero have been taken into account (see Figure 5). The first three eigenfrequencies have been found (see Table 1) using the new FGM beam finite element for modal analysis. Only one our new finite element was used for each beam. The same problem has been solved using a very fine mesh – 24000 of 2D PLANE42 elements of the FEM program ANSYS [8]. The average relative difference Δ[%] between eigenfrequencies calculated by our method and the ANSYS [8] solution has been evaluated.

Table 1. Eigenfrequency of the microactuator

Eigenrequency *f* (Hz)	New finite element	ANSYS	Δ (%)
1st	163 741	163 542	0,12
2nd	234 118	233 941	0,07
3rd	524 309	523 139	0,22

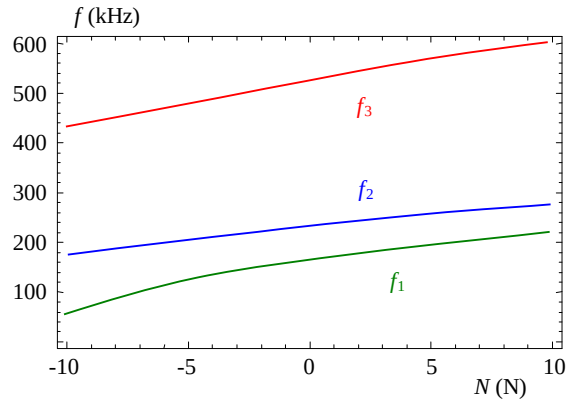

Fig.5. Effect of the axial force on the eigenfrequency

5 CONCLUSIONS

Our new beam finite element has been used in modal analysis of the FGM micro-electro-mechanical system. Continuous transversal and longitudinal variation of material properties have been considered. Effect of large axial force (which can be induced by electro-thermal or electromagnetic loads) on the eigenfrequency was studied. Shear force effect was considered as well - the average value k^{sm} of the shear correction function $k^s(x)$ has been applied.

The obtained results have been studied and compared with results obtained using a very fine mesh of the FEM program ANSYS. The main additions of our new approach are:

- By new finite element can be analyzed not only a single beams, but we can analyze the FGM beam structures – e.g. Micro-Electro-Mechanical Systems (MEMS) made of FGMs with spatial variation of material properties.
- The axial force, induced by working loads in FGMs micro-electro-mechanical sensor or actuator, can change their dynamic properties significantly.
- Our new finite element is very effective and accurate.

Acknowledgement:

This paper was supported by the Slovak Research and Development Agency under the contract No. APVV-0280-10 Integrated Analysis of the Solar Power. This paper was also supported by project ITMS26240220060 Crisis and by Grant Agency KEGA, project No. 015STU-4/2012.

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Received 8 September 2012

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