

# COMPENSATION FERROMETER MAGNETIZING PROCESS CONTROL

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This paper deals with the model of compensation ferrometer magnetizing equipment suitable for magnetizing process control. The magnetizing equipment model provides voltages in Rogowski-Chattock potentiometer and voltage winding that must be controlled on the base of input voltages  $i_e$  magnetizing voltage and compensation voltage. This model uses simple three parameters model of the specimen hysteresis loop in saturation state that is suitable for fast real-time parameters identification.

Keywords: compensation ferrometer model, magnetizing process

## 1 INTRODUCTION

Precise and fast measurement of open specimen soft magnetic materials properties at AC magnetization requires efficient magnetizing process control. Two conditions must be fulfilled in the case of compensation ferrometer measurements. The voltage induced in the measuring winding must be sinusoidal (form factor  $1.11 \pm 1\%$  IEC 404), and the voltage induced in the Rogowski-Chattock potentiometer must be zero. Development of efficient magnetizing process control requires the model of compensation ferrometer magnetizing equipment. Currently, there is not available work that deals with compensation ferrometer model or simultaneous compensation and correction control based on model.

The simplified model of the magnetizing equipment for the compensation ferrometers is shown in Fig. 1. The compensation method uses close magnetic circuit consisting of measured specimen (black) and yoke (grey).

The magnetic circuit is completed by the magnetizing winding (MW), compensating winding (CW) and voltage winding (VW). Special part of the magnetizing equipment is a flat U shape coil with non-magnetic core called Rogowski-Chattock potentiometer (RCP). The RCP encloses the measured region, [1].

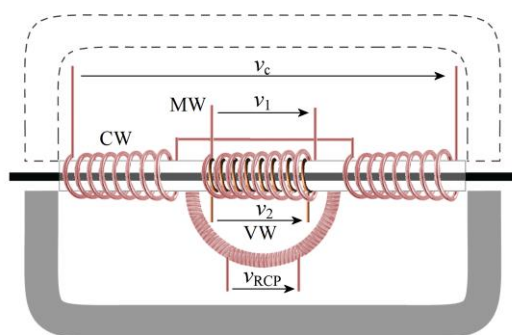


Fig. 1. Magnetizing equipment

The magnetizing equipment model for control development must be capable to describe voltage induced to the RCP and distortion of the voltage induced in the measuring winding in state of the specimen saturation. The magnetizing and compensating winding are powered from

voltage source, so magnetizing and compensating voltages are model inputs. Outputs important for magnetizing process control are voltages in RCP and VW.

## 2 MAGNETIZING EQUIPMENT MODEL

### 2.1 Assumptions and symbols

Presented model is derived with these assumptions. It is sufficient for the system behaviour modelling. Exact fitting is not required.

1) The magnetic flux density is perpendicular to the cross-section. With respect to the geometry of the magnetizing equipment this is possible assumption.

2) The magnetic flux density is not a function of position in the cross-section. This is true for the thin specimen when magnetic skin-effect is negligible.

3) The magnetic field strength in the air is considered to be the same as in the specimen because dominant tangential components on boundary between the air and the specimen are identical. The magnetic flux in the air is taken into account because all windings are not wound tightly on the measured specimen. The air cross-section is sufficiently small for this assumption.

4) Specimen and air cross-sections are constants. This neglects the magnetostriction.

5) The magnetic circuit consists of three regions. Inside these regions magnetic field strength is constant. The first region is specimen, the second region is air gap between specimen and yoke and the third one is yoke.

6) The magnetic flux is same in all three regions.

7) The specimen is considered in the saturation state, where it is hard to save required magnetizing conditions.

8) The yoke is not in the saturation state, because it has much larger cross-section than specimen. [2]

All used symbols are defined in Table 1.

Table 1. List of symbols

$v_1, v_c$ (V)	magnetizing, compensating voltage
$v_{RCP}, v_2$ (V)	voltage induced in RCP, VW
$R_1, R_c$ ( $\Omega$ )	MW, CW resistance
$L_1, L_c, L$ (H)	MW, CW, common differential inductance
$N_1, N_c, N_2$	number of MW, CW, VW turns
$i_1, i_c$ (A)	MW magnetizing, CW compensating current
$i_{1n}, i_{cn}$ (A)	transformed currents
$H, H_Y$ (A/m)	specimen, yoke magnetic field strength

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$H_c$ (A/m)	specimen model parameter
$H_Y$ (A/m)	magnetic field strength computed as in compensation ferrometer
$\Phi$ (Wb)	magnetic flux
$B, B_Y$ (T)	specimen, yoke magnetic flux density
$B_a$ (T)	specimen model parameter - flux density
$l_1, l_Y, l_G$ (m)	specimen, yoke, air gap length
$d$ (m)	distance between RCP ends
$S_G, S_Y, S_M$ (m <sup>2</sup> )	air gap, yoke, specimen cross-section
$S_A, S_c$ (m <sup>2</sup> )	air cross-section, MW (CW) cross-section
$k$ (-)	MW to CW magnetic bond coefficient
$k_a$ (m/A)	specimen model parameter
$k_r$ (H)	RCP constant
$s$ (-), $s_Y$ (-)	specimen, yoke model parameter
$t$ (s)	time
$\mu_0, \mu_Y$ (H/m)	vacuum, yoke permeability
$R_m$ (H <sup>-1</sup> )	magnetic reluctance
$U_r$ (A)	magnetic voltage outside measured region
$\alpha, \beta, \gamma, \delta, \varepsilon, \psi$	yoke model parameters
$a, b, \varphi, x_c, y_c$	main yoke model parameters

## 2.2 Equipment model

The model of compensation ferrometer magnetizing equipment consists of the following equations (1)-(10).

$$-v_1 + R_{l_1} i_1 + N_1 L_1 \frac{di_1}{dt} + N_1 L_c \frac{di_c}{dt} = 0 \quad (1)$$

$$-v_c + R_c i_c + k N_c L_c \frac{di_c}{dt} + k N_c L_1 \frac{di_1}{dt} = 0 \quad (2)$$

These equations are differential algebraic equations (DAE) because the Jacobian is singular. The differentiation index is one, [2].

Inductances are defined according (3). These differential inductances are nonlinear function of  $H$ .

$$L_1 = \frac{d\Phi}{dH} \frac{\partial H}{\partial i_1} \quad L_c = \frac{d\Phi}{dH} \frac{\partial H}{\partial i_c} \quad (3)$$

The magnetic flux is obtained from model of specimen hysteresis loop based on hyperbolic tangent, [3],[4]. This hysteresis loop model has three parameters only:  $B_a$ ,  $k_a$  and  $H_c$ . The parameter  $B_a$  corresponds to the magnetic flux density maximum. The parameter  $H_c$  is coercivity and the parameter  $k_a$  corresponds to slope of the hysteresis loop. The variable  $s$  distinguishes between ascendant or descendant part of hysteresis loop. Only values 1 and -1 are allowed for the  $s$  variable, [3],[4].

$$\Phi = S_M [\mu_0 H + B_a \tanh(k_a (H + H_c s))] + S_A \mu_0 H \quad (4)$$

$$s = -\text{sign}\left(\frac{dH}{dt}\right) = -\text{sign}\left(\frac{\partial H}{\partial i_1} \frac{di_1}{dt} + \frac{\partial H}{\partial i_c} \frac{di_c}{dt}\right) \quad (5)$$

Magnetic field strength is given by implicit equation. The  $H_Y$  is expressed from the yoke hysteresis loop model.

$$H l_1 + \frac{l_G}{\mu_0 S_G} \Phi + H_Y l_Y = N_1 i_1 + N_c i_c \quad (6)$$

Suitable model for the yoke is ellipse, where  $s_Y$  distinguishes between ascendant or descendant part of the hysteresis loop. Parameters  $a$ ,  $b$  and  $\varphi$  are the semi-major,

semi-minor axes and rotation angle, respectively. When center of the ellipse is  $x_c = 0$ ,  $y_c = 0$ , then parameter  $a$  corresponds to the maximal magnetic field strength in yoke, parameter  $b$  corresponds to remanent induction and  $\tan \varphi$  is related to the amplitude permeability. [2]

$$H_Y = \frac{-\beta B_Y - \delta + s_Y \sqrt{(\beta B_Y + \delta)^2 - 4\alpha(\gamma B_Y^2 + \varepsilon B_Y + \psi)}}{2\alpha} \quad (7)$$

$$s_Y = \text{sign}\left(\frac{dH_Y}{dt}\right) = -s \cdot \text{sign}\left(\frac{dH}{dt}\right) \quad (8)$$

$$\alpha = a^2 \sin^2 \varphi + b^2 \cos^2 \varphi \quad \beta = 2(b^2 - a^2) \sin \varphi \cos \varphi$$

$$\gamma = a^2 \cos^2 \varphi + b^2 \sin^2 \varphi \quad \delta = -2\alpha x_c - \beta y_c \quad (9)$$

$$\varepsilon = -\beta x_c - 2\gamma y_c \quad \psi = \alpha x_c^2 + \beta x_c y_c + \gamma y_c^2 - a^2 b^2$$

The magnetic flux density inside the yoke is obtained from common magnetic flux.

$$B_Y = \frac{\Phi}{S_Y} \quad (10)$$

Another option for the yoke is linear model. This lead to change from (6) to (11) where the magnetic reluctance is used.

$$H l_1 + R_m \{S_M [\mu_0 H + B_a \tanh(k_a (H + H_c s))] + S_A \mu_0 H\} = N_1 i_1 + N_c i_c \quad (11)$$

$$R_m = \frac{l_G}{\mu_0 S_G} + \frac{l_Y}{\mu_Y S_Y} \quad (12)$$

Differential inductances could be significantly simplified in case of the linear yoke model.

$$L_1 = \frac{N_1 \{ \mu_0 S_c + S_M B_a k_a [1 - \tanh^2(k_a (H + H_c s))] \}}{l_1 + \mu_0 R_m S_c + R_m S_M B_a k_a [1 - \tanh^2(k_a (H + H_c s))]} \quad (13)$$

$$L_c = L_1 N_c / N_1 \quad (14)$$

$$S_c = S_M + S_A \quad (15)$$

The variable  $s$  has simple representation, too.

$$s = -\text{sign}\left(N_1 \frac{di_1}{dt} + N_c \frac{di_c}{dt}\right) \quad (16)$$

The outputs are given by (17)-(21) in both cases (linear or elliptic yoke model).

$$B = \mu_0 H + B_a \tanh(k_a (H + H_c s)) \quad (17)$$

$$v_2 = N_2 S_M \frac{dB}{dt} \quad (18)$$

$$v_{RCP} = \frac{2k_r}{\pi d} \frac{d(N_1 i_1 - Hd)}{dt} \quad (19)$$

$$H_Y = N_1 \frac{i_1}{d} \quad (20)$$

$$U_r = N_1 i_1 + N_c i_c - Hd \quad (21)$$

The compensation ferrometer magnetizing equipment model input variables are the voltages  $v_1$  and  $v_c$ . The outputs important for magnetizing process control are the voltages  $v_2$  and  $v_{RCP}$ . However the core model equations (1) (2) are DAE for the currents  $i_1$  and  $i_c$ .

Complete model solving proceeds in these steps: computation of DAE consistent initial condition, DAE solving, outputs computations. Solver for the DAE system is ode15i (MATLAB) for example. It solves fully implicit differential equation using the variable order backward differentiation formula method. Other DAE solvers are also applicable. Inside the DAE solver is necessary to numerically solve the implicit equation (6) or (11) and equations for variable  $s$  (and  $s_Y$ ). Numerical differentiation is used where is needed. [2]

The model output signals content unwanted higher harmonic components. This is caused by sharp edges in the signals when  $s$  changes. This is the reason for filtering of  $i_1$ ,  $i_c$  and  $B$  by linear phase FIR filter with cut-off frequency ten times higher than magnetizing frequency.

### 3 MODEL TRANSFORMATIONS

The compensation ferrometer magnetizing equipment model equations are not in a friendly form. Transformations to another form are described in this section.

It is necessary to express derivatives of the currents from (1) and (2). Let's substitute  $di_1/dt$  expressed from (1) to (2). The result is algebraic equation. Equations (1) and differentiation of the algebraic equation with respect to the time form a set of equations from those currents derivatives are expressed.

Obtained equation set contain derivatives of the both input voltages and does not contain  $v_c$ . They are not symmetrical as the origin equation set. Note that the algebraic equation does not contain inductances. Repeat the procedure from previous paragraph with  $di_c/dt$  and exchanged (1) and (2) lead to the second equation set with expressed currents derivatives.

The symmetrical equation set that contain  $v_c$  is obtained by summation of appropriate equations from the first and the second equation set.

Equation (14) is valid for linear and elliptic yoke model. This allows introducing common differential inductance  $L$ .

$$L_1 = LN_1 \quad L_c = LN_c \quad (22)$$

Transformation below introduces new currents  $i_{1n}$  and  $i_{cn}$ . This transformation remove dependency on the input voltages derivatives ( $dv_1/dt$  and  $dv_c/dt$ ), [5].

$$i_{1n} = i_1 - \frac{N_c^2 k}{R_c N_1^2 + N_c^2 k R_1} v_1 + \frac{N_c N_1}{R_c N_1^2 + N_c^2 k R_1} v_c \quad (23)$$

$$i_{cn} = i_c + \frac{N_c k N_1}{R_c N_1^2 + N_c^2 k R_1} v_1 - \frac{N_1^2}{R_c N_1^2 + N_c^2 k R_1} v_c \quad (24)$$

Note that transformation does not contain inductances again. The result is equation set for the new currents derivatives. Due to the symmetry of this equation set following equations are valid.

$$\frac{di_{1n}}{dt} = \frac{R_c^2 N_1^2 v_1 + R_c N_c R_1 N_1 v_c - R_c (R_1 R_c N_1^2 + R_1^2 N_c^2 k) i_{1n}}{L (R_c N_1^2 + N_c^2 k R_1)^2} \quad (25)$$

$$i_{1n} = \frac{N_c k R_1}{R_c N_1} i_{cn} \quad (26)$$

Result core equation (25) is in ordinary differential equation form. These results could be substituted to other equations. The variable  $s$  has simple representation in this new current.

$$s = -\text{sign}\left(\frac{di_{1n}}{dt}\right) \quad (27)$$

The magnetizing equipment model is nonlinear first order system for transformed current (not DAE). It could be solved by an explicit Runge-Kutta (4, 5) formula - ode45 (MATLAB). Other solving steps remain the same.

The output voltage  $v_2$  depend on  $v_1$ ,  $v_c$  and  $i_{1n}$ . The output voltage  $v_{RCP}$ , moreover, depend on derivatives of  $v_1$  and  $v_c$ . The model is the first order system with two inputs and outputs, but it can not be controlled by one input only because both outputs depends on inputs directly. The outputs are coupled and could not be decoupled in this form. Model reference trajectory could be computed from the magnetizing conditions and equations for the outputs.

### 4 RESULTS

The next figures show results of the magnetizing equipment model with linear yoke model computations. Differences in results from elliptic yoke model version are negligible. Equipment model parameters were estimated from real yoke. The specimen hysteresis loop model parameters were estimated from measured hysteresis loop.

The measured material is silicon steel EO10 - sheets 0.5 m x 0.5 m, thickness 0.35 mm. The material was measured by compensation ferrometer KF9a on frequency 50 Hz. The magnetic flux density amplitude was 1.6 T.

The model parameters are:  $v_1 = 7.6\sin(2\pi \cdot 50t)$  V,  $v_c = 8.6\sin(2\pi \cdot 50t)$  V,  $N_1 = N_c = 72$ ,  $N_2 = 10.8$ ,  $R_1 = R_c = 16.58 \Omega$ ,  $B_a = 1.6$  T,  $k_a = 0.123$ ,  $H_c = 25.8$  A/m,  $S_M = 166$  mm<sup>2</sup>,  $S_A = 5S_M$ ,  $l_1 = 0.3$  m,  $l_G = 2$  mm,  $l_Y = 0.5$  m,  $\mu_Y = 5e3\mu_0$ ,  $S_G = S_Y = 0.1$  m<sup>2</sup>,  $k = 0.99$ ,  $d = 0.1$  m,  $k_r = 6.3$   $\mu$ H.

The model of the magnetizing equipment has high degree of conformity with the measured signals. This could be seen in Fig. 4.

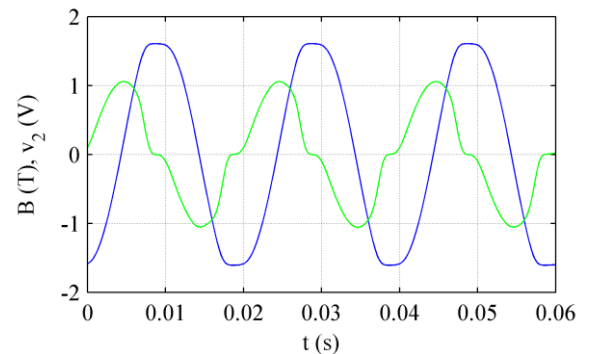


Fig. 2. Voltage  $v_2$  and magnetic flux density  $B$

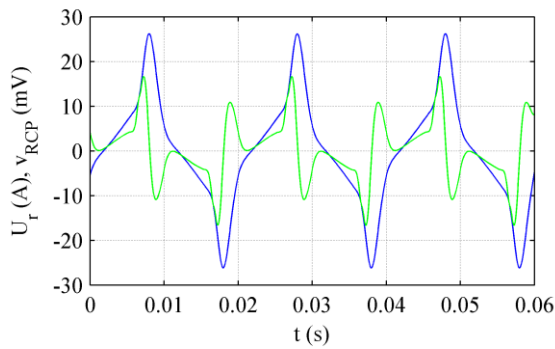


Fig. 3. Voltage  $v_{RCP}$  and magnetic voltage  $U_r$

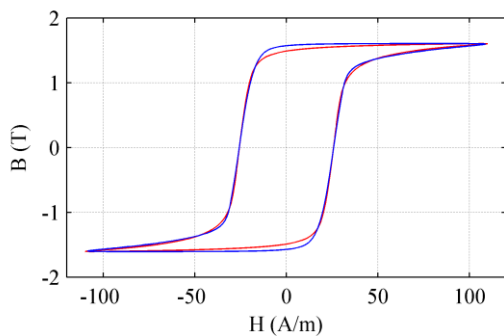


Fig. 4. Measured and simulated hysteresis loop

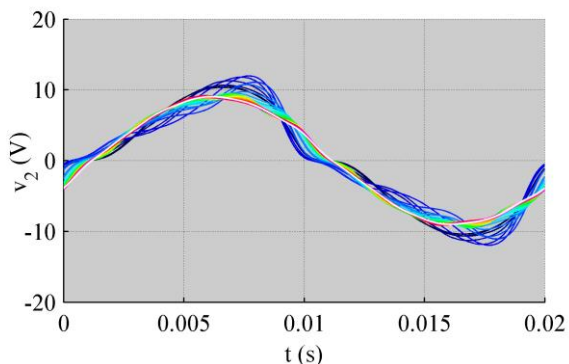


Fig. 5. Correction process

The magnetic flux density waveform correction process simulation is in Fig. 5. Iterations are colored from black to white, where black is the first iteration and white is the last iteration. Used algorithm was published in [6]. Compensation was not used ( $v_c = v_1$ ) for this simulation.

## 5 CONCLUSION

The model of the compensation ferrometer magnetizing equipment is capable to describe voltage induced to the RCP and distortion of the voltage induced to the voltage winding in the state of specimen saturation, where is hard to satisfy magnetizing conditions. This model works in given operation point (magnetic flux density amplitude). It uses three parameters specimen model that ensures possibility of real-time adaptation on measured specimen. The magnetizing process control based on non-linear model of the compensation ferrometer equipment

with the real-time adaptation on measured specimen is promising. Important advantage of this magnetizing process control is possibility of all conditions control as one complex problem. The current state of the model allows the first estimation of the input voltages.

The future work will be oriented on research and development of suitable control algorithms for magnetizing processes that ensure effective high speed, stable and precise measurement. Possibilities of exact feedback linearization in combination with iterative learning control will be studied.

## Acknowledgement

Research described in the paper was supported by Department of Circuit Theory, Faculty of Electrical Engineering, Czech Technical University in Prague and the Grant Agency of the Czech Technical University in Prague, grant No. SGS14/191/OHK3/3T/13 Advanced Algorithms of Digital Signal Processing and their Applications.

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Received 30 November 2015

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