MAGNETIC MOMENT OF PERMANENT MAGNET MEASUREMENT WITH NONLINEAR LEAST SQUARES FITTING METHOD

Michał Nowicki* — Dorota Jackiewicz — Roman Szewczyk****

In the paper, a procedure for magnetic moment measurement, utilizing single sensitive magnetometer and linear table is described. The object under test, for example a permanent magnet, is positioned with its main axis along the sensitive axis of the magnetometer's moving probe, and the magnetic field strength is recorded, as well as the probe precise position. The results of measurement are then processed in the Matlab software, and the non-linear least squares curve fitting is done using the Levenberg–Marquardt algorithm. The fitting is performed for the magnetic dipole magnetic field strength versus distance model. Thus it is possible to measure both the object's magnetic moment and distance from the magnetometer.

Keywords: magnetic moment, magnetoresistive sensor, Levenberg-Marquardt algorithm

1 INTRODUCTION

Measurement of magnetic moment is a convenient way to test permanent magnet materials. Other values such as operating flux density (B_d), operating field strength (H_d), coercive force (H_c), residual flux density (B_r), and maximum energy product (BH_{max}) can be derived from the measured moment value. Although this method is not as accurate as hysteresisgraph measurement, the measurement process is easy, values are useful and reliable, and equipment cost is substantially less [1].

Fluxgate sensor gaussmeter probe is widely used for magnetic moment measurements. The measurement is done in a fixed distance from the tested object, based on which, and simple magnetic dipole equation, the magnetic moment is calculated. This method, however simple, is not as precise as the more expensive Helmholtz coil and integrator method, taking also into account the magnet geometry [1,2]. The distance of the sensor from the magnet, and the ambient magnetic field are the main factors in the uncertainty of the measurement.

In the paper, a procedure for magnetic moment measurement, utilizing single sensitive magnetometer is described. The object under test, for example a permanent magnet, is positioned with its main axis along the sensitive axis of the magnetometer's moving probe, and the magnetic field strength is recorded, as well as the probe precise position. The results of measurement are then processed in the Matlab software, and the non-linear least squares curve fitting is done using the Levenberg–Marquardt algorithm [3]. The fitting is performed for the magnetic dipole magnetic field strength versus distance model. The important factor is that the distance of the first measurement point from the measured object is also treated as unknown, and thus not essential to perform the procedure. The initial (without measured object) local magnetic field distribution is also recorded. Therefore only the information of the magnetic field strength value in measurement points, and the distance between them, is needed for the magnetic moment measurement.

To evaluate the proposed method, a laboratory test stand was constructed. The first part of the test stand consist of the Helmholtz coil and precise fluxmeter setup for the precise, reference magnetic moment measurement. The second part consists of magnetoresistive magne-tometer, and a linear table for probe manipulation. The third part is the simple method MR sensor magnetometer fixed in the given distance from the magnet, for comparison purpose.

2 MEASUREMENT METHODS

The measurements of the sample permanent magnets magnetic dipole moment were performed on three separate test stands, two of which are widely used in industry and laboratories. These two were treated as the reference measurements. The third measurement stand was specially constructed for the newly developed method, utilizing the non-linear least squares fitting method of the results.

2.1 Simple magnetometer measurement

The simplest and most common way to measure the magnetic dipole moment is the single magnetometer utilization. The magnetic induction absolute value in distance r from the magnetized object M with angle φ to the object axis is

$$\left|\vec{B}\right| = \frac{\mu_0 M}{4\pi r^3}(1 + 3cos^2\varphi_m)^{\frac{1}{2}} \quad (1)$$

The magnetometer head, *eg* MR or Fluxgate sensor based, is positioned on the axis of the measured object, in the known distance. The moment is then calculated with the formula (simplified from formula 1) (2) or (3) (case dependent), where the distance r is given in metres, the magnetic induction B in μT, and the resultant magnetic moment is given in SI derived unit Am^2.

Case 1 – Magnetometer probe on the axis of the measured magnet (Fig. 1).

$$m = 5Br^3 \quad (2)$$

* Industrial Research Institute for Automation and Measurements, Warsaw, Poland, nowicki@mchtr.pw.edu.pl, ** Institute of Metrology and Biomedical Engineering, Warsaw University of Technology, Warsaw, Poland, d.jackiewicz@mchtr.pw.edu.pl, szewczyk@mchtr.pw.edu.pl

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where B – is the measured induction value, r – is the distance between the centre of the magnet and the magnetometer probe.

Case 2 - parallel probe and magnet setup

$$m = 10Br^3 \tag{3}$$

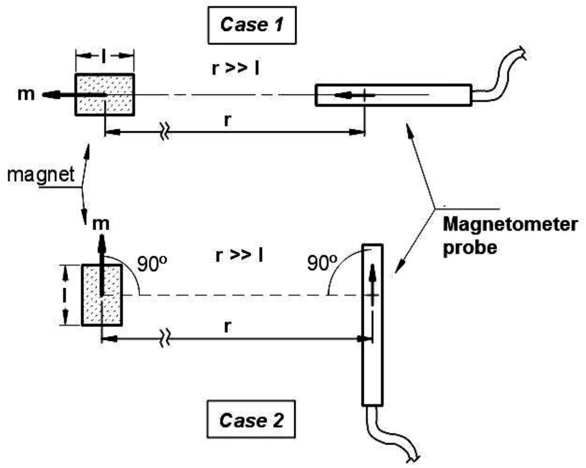


Fig. 1. The magnetometer and magnet positions. Case 1 – magnetometer probe along the dipole axis, Case 2 – parallel probe and magnet setup. Case 1 is two times more sensitive for given magnetic dipole *m* and distance *R*, [2]

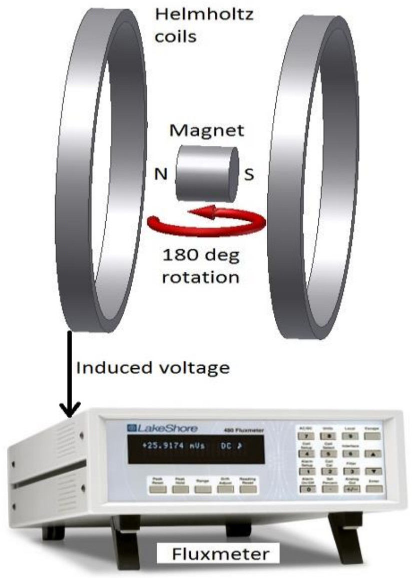


Fig. 2. The schematic diagram of the Helmholtz coils magnetic moment measurement method.

The measurement does not depend on the shape or homogeneity of the magnet under test, but is heavily influenced by the proper magnetometer positioning. The "magnet under test" can be almost any magnetized object, such as machine, engine, or even a satellite.

In Case 1, on which the magnet and the probe are coaxial, sensitivity is double than in Case 2, thus Case 1 results the more adequate configuration when it is feasible to use. The distance (r) must be about 10 times the bigger magnet dimension. In such a way the measurement should be almost independent from the magnet shape and homogeneity. To improve the measurement and dismiss the Earth magnetic field, it is convenient to perform two measurements with the magnet 180º rotated in respective of each. Then, one reading is subtracted from the other and the result is divided by 2 to get B. Conversely, one can zero the magnetometer readings without the magnet, and then take measurement with the magnet positioned in desired distance r.

In this paper, a sensitive triaxial magnetoresistive magnetometer Honeywell 2300 was used. It has 7 nT resolution and ± 200 µT range. The investigated magnets were placed in desired distance r from the magnetometer, with their dipole axis coaxial with the X axis of the magnetometer. The Earth's field was zeroed, and values of magnet induced B were taken for various distances r.

2.2 Helmholtz coils method

Helmholtz coils are a pair of thin, parallel and identical coils connected in series, placed in a distance equal to their radius. They are commonly used to provide uniform magnetic field within them. This concept can be reversed however, to use the coils as flux-sensing coils, to measure the open circuit magnetization with an integrating fluxmeter. When used with a Helmholtz coil, a fluxmeter provides a value for the magnetic moment of a test sample when the magnet is placed between the coils in alignment with the coil axis and rotated 180 degrees or withdrawn [4]. It should be stressed, that in the case of sample rotation the sensitivity (and fluxmeter readings) is two times higher than in the case of sample withdrawal [5].

The voltage induced in the coils is integrated in the fluxmeter, and the magnetic moment is calculated on the basis of the coils constant C. The open-circuit magnetization is related to the time-integrated voltage by

$$4\pi M_0 = \frac{C}{V} \int E\mathrm{d}t \tag{4}$$

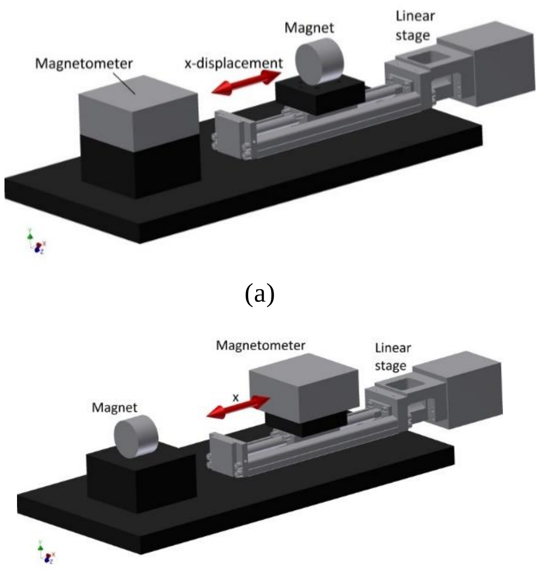
Where V is the sample volume, C is the coil constant and E is the voltage induced in the coils by sample rotation. The coil constant can be calculated using known Helmholtz coils formulas, or measured empirically using appropriately sensitive magnetometer.

In this paper, a self-made pair of Helmholtz coils were used, with the constant C calculated and measured as 1 cm. The high-quality Lakeshore model 480 fluxmeter was connected to the coils output. Investigated magnets were rotated 180° in the centre of the coils, and the fluxmeter readings were recorded.

Due to the high quality measuring equipment and well-established measurement method, the magnetic moment values obtained for the measured samples are regarded as reference values for the other methods.

2.3 Nonlinear least squares fitting method

The method proposed in this paper is in fact development of the first and simplest method, utilizing single magnetometer. It was noticed, that the magneto-meter method, however basic and utilizing inexpensive equipment, can be corrected. One of the main error sources is the measurement of distance between the magnet and the magnetometer sensing element. By taking multiple *B* measurements in different distances from the measured object, and fitting the results to the magnetic dipole equation (1), one can calculate the magnetic dipole moment value. Furthermore, the critical information are the magnetic induction values in the measurement points, and the distances between the measurement points. The distance between the magneto-meter and the measured object can be treated as unknown, and calculated *together* with the magnetic moment value. This can be potentially used for detection and localization of hidden ferromagnetic objects.



(a)

(b)

Fig. 3. The magnetometer and magnet setup for magnetic moment measurement with nonlinear least squares method: (a) - fixed magnetometer, measured object moving on the linear table (better accuracy), (b) - fixed measured object, magnetometer moving on the linear table (ability to measure cumbersome objects).

To test the method, special measurement stand was constructed. It consisted of the linear table with displace-ment measurement, and sensitive magneto-resistive mag-netometer Honeywell HMR2300. The whole system was automated and worked under the Labview based software, which allowed to take the measurement along the designated line, with a preset spacing between the measurement points.

The sample magnets can be placed still on the axis of the moving magnetometer (Fig. 3b), or vice versa – the magnetometer can be fixed, and measured object placed on the linear table (Fig 3a). The second case allows for better precision of the measurement, as it is easier to zero the influence of external magnetic fields. The first case however, potentially allows for measurement of bigger objects, even cars or satellites.

After the measurement, the induction values and measurement point coordinates are processed in the Matlab software. The data is fit using the nonlinear least squares method to the equation derived from the (1) and (2)

$$y = \frac{\frac{m}{5}}{(x+b)^3} + c + dx \qquad (5)$$

where: y – magnetic induction values in measurement points in μT, m - magnetic moment in Am^2, x – distance coordinate in m, first measurement point is 0, b – distance offset (distance of the first measurement point from the object), c – constant value of magnetic induction (it allows to use the method even without zeroing the magnetometer for Earth's field), d - magnetic gradient replacement constant (allows to use the method, however with greater uncertainty, in external magnetic field gradients).

The Levenberg-Marquardt algorithm was chosen as the most effective, together with preliminary robust LAR method.

3 MEASUREMENT RESULTS

Three different magnet samples were chosen for the measurements, that is:

- F30 ferrite permanent magnet, cylinder shaped, Ø8x10 mm
- Alnico permanent magnet, cylinder shaped, Ø10x10 mm
- NdFeB permanent magnet, sphere shaped, Ø15 mm

Each of the samples was measured in a couple of different distances with method 1, then with method 2 (Helmholtz coils) for reference, and finally with the 3 method. The sample results from the new measurement method, namely the curve fit, can be seen in Fig. 4.

The comparison of the measurement results is in Table 1. There is a significant difference between the results of the first method with different magnetometer-magnet distances, which highlights the basic limitations of this measurement. There is a bigger difference however, between the results of the first two methods and the new one.

The model (5) fits the measurement points, and, conversely, on the basis of the magnetic moment derived from the new method and equation (2) one can accurately calculate magnetic induction value in the given distance from the measured magnet.

It even seems that this value is better than the obtained from the first two methods. It should be highlighted however, that the fluxmeter- Helmholtz coils setup was set accordingly to the manufacturer manual, and the results of the first two methods are similar.

Table 1. Measurement results

Sample	Simple magnetometer method (Am^2) Distance *r* (m)			Helmholtz (Am^2)	Least sq. (Am^2)
	0.5	0.4	0.3		
Ferrite	0,150	0,153	0,155	0,159	0,134
Alnico	0,138	0,128	0,125	0,135	0,114
-	0.9	0.7	0.5	-	-
NdFeB	1,786	1,715	1,700	1,769	1,295

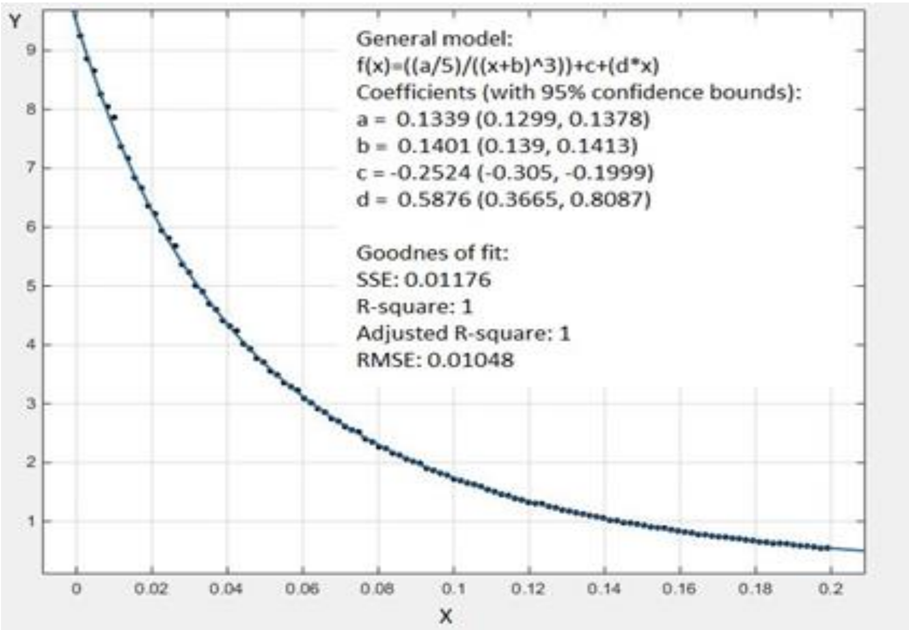


Fig. 4. The measurement results (black dots) and the Levenberg-Marquardt curve fit, *a* - is the parameter of measured magnetic moment, *b* - is the distance from the object.

On the other hand, there was no standard sample of *known* magnetic moment used in the investigation. Thus, it is currently too early to define the error level of the new method.

4 CONCLUSIONS

The new method of magnetic moment measurement is presented. It is based on relatively inexpensive equipment (in comparison with the fluxmeter-Helmholtz coils method), and the results are more consistent than in the simple magnetometer method. The measurements of the magnetic moment were done for the Alnico, NdFeB and F30 ferrite permanent magnets. High reproducibility of the results was achieved. On the other hand, there was significant difference between the new method and the reference ones. The new method however, allows to measure the distance from the unknown object together with it magnetic moment, which may prove useful in hidden objects detection [6]. Additionally, with proper scaling, the proposed method allows for magnetic moment measurement of larger and more complicated objects, for which the Helmholtz coil method would be impractical [2,4].

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