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NUMERICAL ANALYSIS OF A 2D VECTOR HYSTERESIS MEASUREMENT SYSTEM UNDER CONSTRUCTION

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This paper deals with the numerical analysis of a vector hysteresis measurement system which is under construction. The aim is to build up a single sheet tester with round shaped specimen. The goal of simulations is to find out the main features of the measurement system. The 3D finite element method (FEM) with tetrahedral mesh has been applied for investigations of the nonlinear eddy current field problem. The characteristic of the magnetic material has been taken into account by the isotropic vector Preisach model. The nonlinearity has been handled by the polarization method and the nonlinear system of equations has been solved by the fixed point technique. The first results are presented in this work.

Keywords: vector hysteresis measurement, vector hysteresis model, finite element method, fixed point technique

1 INTRODUCTION

Measurement of 2D vector hysteresis characteristics is the aim of research to build up the models of vector hysteresis characteristics and to make the Computer Aided Design software more accurate for the simulation of different magnetic equipments of a wide range.

This paper summarizes the preliminary numerical studies of a round shaped rotational single sheet tester (RRSST) system which is under construction in our laboratory. The RRSST system has been chosen because it is easy to get the yoke and it is easy to manufacture the specimen, moreover this arrangement has some advantages as it is presented in [2].

2 THE MEASUREMENT SYSTEM

The measurement system is under construction. The rotational single sheet tester is an induction motor with a 36 slots stator core whose rotor has been removed (see in Fig. 1). The diameter of the round shaped specimen can be about 80-81 mm. The original winding of the motor has been changed to a special two phase one to realize two orthogonal magnetic fields inside the specimen. The magnetic field intensity can be generated by two current generators controlled by the computer which simultaneously collects the measured data. The *B*-coils (measuring flux density) are slipped into holes of the specimen, and the *H*-coils (measuring field intensity) are placed on the surface of the material. The measurements are performed under National Instruments LabVIEW environment.

3 FEM MODEL OF THE MEASUREMENT SYSTEM

3.1 Defining the problem

Eddy currents inside the core can be neglected, because it is made of laminated sheets. It is supposed that there is a static but time varying magnetic field inside the stator core and in the air region, and there is an eddy current field problem inside the analyzed specimen.

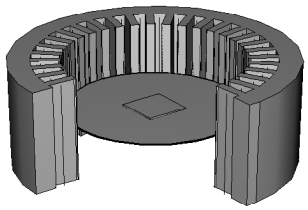

Fig. 1. The RRSST and the specimen under investigation (the special two phase winding is not shown here, see [2])

The static but time varying magnetic field in the eddy current free region and the quasi-static magnetic field in the eddy current region are described by Maxwell's equations

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}_0,\ \nabla\cdot\boldsymbol{B}=0,\ \nabla\cdot\boldsymbol{J}_0=0,\ \boldsymbol{H}=\boldsymbol{B}/\mu\,, \tag{1}$$

and

$$\begin{aligned}&\nabla\times\boldsymbol{H}=\boldsymbol{J}_0+\sigma\boldsymbol{E},\ \nabla\times\boldsymbol{E}=-\dot{\boldsymbol{B}},\ \nabla\cdot\boldsymbol{B}=0,\\&\nabla\cdot(\boldsymbol{J}_0+\sigma\boldsymbol{E})=0,\ \boldsymbol{H}=B\{\boldsymbol{B}\}:=\boldsymbol{B}/\mu_{FP}+\boldsymbol{R}_{FP},\end{aligned} \tag{2}$$

where $\boldsymbol{H}$, $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{J}_0$, μ and σ are the magnetic field intensity, the electric field intensity, the magnetic flux density, the source current density, the permeability and the conductivity, respectively. The permeability is constant in the static magnetic field region, $\mu=\mu_0$ in air, $\mu=1000\mu_0$ in the stator core. The last equation in (2) is the linearized form of the nonlinear and vector hysteretic constitutive relation according to the *H*-scheme of the polarization method. The constant μ_{FP} is the average value of the minimum and the maximum value of permeability, ie $\mu_{FP}=(\mu_{\min}+\mu_{\max})/2$, and $\boldsymbol{R}_{FP}$ is the nonlinear residual term. The 2D isotropic vector Preisach model has been used in the *x*-*y* plane of specimen, and $\mu_z=\mu_{\max}$, ie $\boldsymbol{R}_{FP}\cdot\boldsymbol{e}_z=0$, where $\boldsymbol{e}_z$ is the unit vector in direction *z*.

3.2 The FEM formulation of the problem

The 3D problem has been solved by FEM. The FEM mesh of the arrangement can be seen in Fig. 2. Only the half of RRSST has been analyzed because of symmetry.

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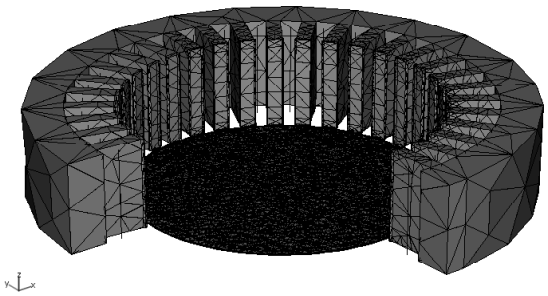

Fig. 2. The geometry of RRSST is meshed by tetrahedral elements

The mesh contains three layers inside the specimen which generates a FEM mesh with 50832 tetrahedral elements and 62739 unknowns (edges).

The edge shape functions have been used to represent the unknown magnetic vector potential [1].

From the third equation in (1) and from the fourth equation in (2) the current vector potential $\boldsymbol{T}_0$ can be introduced to represent the known exciting current,

$$\nabla \times \boldsymbol{T}_0 = \boldsymbol{J}_0 , \tag{3}$$

and it is represented by first order vector shape functions. This can be calculated as the line integral of the magnetic field intensity in free space along every edge of the mesh.

According to the second equation in (1), the so called $\boldsymbol{A}$-formulation can be used in the eddy current free region, ie the magnetic flux density can be represented by the magnetic vector potential $\boldsymbol{A}$ as $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. Using the magnetic vector potential, the impressed field quantity in (3), and the constitutive relation in (1) leads to the partial differential equation

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A} \right) = \nabla \times \boldsymbol{T}_0 . \tag{4}$$

According to the third equation in (2), a magnetic vector potential $\boldsymbol{A}^*$ can be introduced in the eddy current region as well, ie $\boldsymbol{B} = \nabla \times \boldsymbol{A}^*$. This is the so called modified magnetic vector potential. Substituting this relation into the second equation in (2) results in the formulation of the electric field intensity $\boldsymbol{E}$, ie $\boldsymbol{E} = -\partial \boldsymbol{A}^* / \partial t$ (the electric scalar potential is equal to zero). Using the modified magnetic vector potential, the impressed field quantity in (3), and the linearized constitutive relation in (2) leads to

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \boldsymbol{A}^* \right) + \sigma \frac{\partial \boldsymbol{A}^*}{\partial t} = \nabla \times \boldsymbol{T}_0 - \nabla \times \boldsymbol{R}_{FP} . \tag{5}$$

The magnetic vector potentials have been represented by first order vector shape functions which divergence is equal to zero, and it is resulting divergence-free magnetic vector potential (Coulomb gauge),

$$\nabla \cdot \boldsymbol{A} = 0, \ \nabla \cdot \boldsymbol{A}^* = 0 . \tag{6}$$

In this situation the fourth equation in (2) has been satisfied automatically.

Applying the modified magnetic vector potential in the eddy current region and the magnetic vector potential in the eddy current free region is the so called $\boldsymbol{A}^* - \boldsymbol{A}$ formulation.

It is supposed that the normal component of the magnetic flux density vanishes on the boundary and on the symmetry plane, which can be prescribed by the boundary conditions

$$\boldsymbol{A} \times \boldsymbol{n} = \boldsymbol{0}, \ \boldsymbol{A}^* \times \boldsymbol{n} = \boldsymbol{0} . \tag{7}$$

The tangential component of magnetic vector potential is continuous along the interface between the specimen and the air region, because they are represented by edge shape functions. This is equal to the continuity of the normal component of the magnetic flux density along the interface. The continuity of the tangential component of the magnetic field intensity is satisfied in a weak sense.

After applying the Galerkin method and some mathematical manipulations, the following system of equations can be obtained in the eddy current free region

$$\int_\Omega \nabla \times \boldsymbol{W}_j \cdot \left(1/\mu \nabla \times \boldsymbol{A} \right) \mathrm{d}\Omega = \int_\Omega \nabla \times \boldsymbol{W}_j \cdot \boldsymbol{T}_0 \, \mathrm{d}\Omega . \tag{8}$$

In the eddy current region the following nonlinear equation can be got

$$\begin{aligned} &\int_\Omega \nabla \times \boldsymbol{W}_j \cdot \left([1/\mu] \nabla \times \boldsymbol{A}^* \right) \mathrm{d}\Omega + \int_\Omega \sigma \boldsymbol{W}_j \cdot \frac{\partial \boldsymbol{A}^*}{\partial t} \mathrm{d}\Omega = \\ &\int_\Omega \nabla \times \boldsymbol{W}_j \cdot \boldsymbol{T}_0 \, \mathrm{d}\Omega - \int_\Omega \nabla \times \boldsymbol{W}_j \cdot \boldsymbol{R}_{FP} \, \mathrm{d}\Omega, \end{aligned} \tag{9}$$

where $[1/\mu]$ is the tensor of elements $1/\mu_{FP}$, $1/\mu_{FP}$, $1/\mu_z$ in the diagonal. The Euler backward scheme has been used to approximate $\partial \boldsymbol{A}^* / \partial t$. After applying the approximating edge elements and the boundary conditions a system of nonlinear equations can be obtained.

3.3 The nonlinear iteration scheme

The above described system of nonlinear equations can be solved by iterative methods. The fixed point technique has been used in this work.

The nonlinear hysteresis characteristic has been linearized by the polarization method, and the nonlinear residual term $\boldsymbol{R}_{FP}$ has been determined by the following iteration algorithm. The constant values in the tensor $[1/\mu]$ means that the left hand side of the resulting system of linear equations is constant and can be written in the form $\mathbf{K}(\mu_{FP})\mathbf{a}^{(k)} = \mathbf{b}(\boldsymbol{R}_{FP}^{(k-1)})$. The right hand side of the system of equations is updated iteratively,

1. $k = 0$, $\boldsymbol{R}_{FP}^{(0)}$ is equal to the value of residual in the last fixed point,
2. $k = k + 1$, setting up the right hand side of the system of equations, and solving it,
3. calculating $\boldsymbol{B}^{(k)}$ and $\boldsymbol{H}^{(k)}$ by the vector hysteresis model using the regula falsi method,
4. updating the residual term, and step back to 2. if

$$\sqrt{\frac{1}{N_h} \sum \left(\boldsymbol{R}_{FP}^{(k)} - \boldsymbol{R}_{FP}^{(k-1)} \right)^2} < \varepsilon \tag{10}$$

is not satisfied. Here ε is a small positive limit, N_h is the number of elements inside specimen.

4 NUMERICAL RESULTS

4.1 Preliminary Study: Magnetically Linear Specimen

First the distribution of the magnetic field intensity inside the motor without the specimen has been simulated in a rotational magnetic field, when the amplitude of exciting currents is 1A and the magnetization is rotating, the result is plotted in Fig. 3. The magnetic field intensity is almost the same in every directions, the relative difference between the minimum and the maximum value is $\sim 0.02\%$.

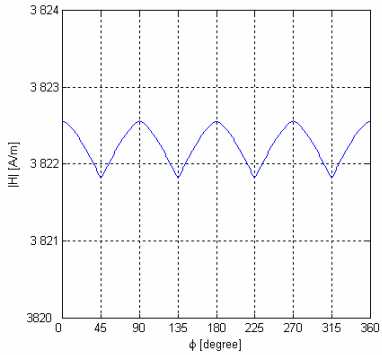

Fig. 3. Amplitude of field intensity in rotational field

The next step is to analyze the effect of eddy currents inside the specimen by increasing the frequency of excitation and keeping its amplitude constant.

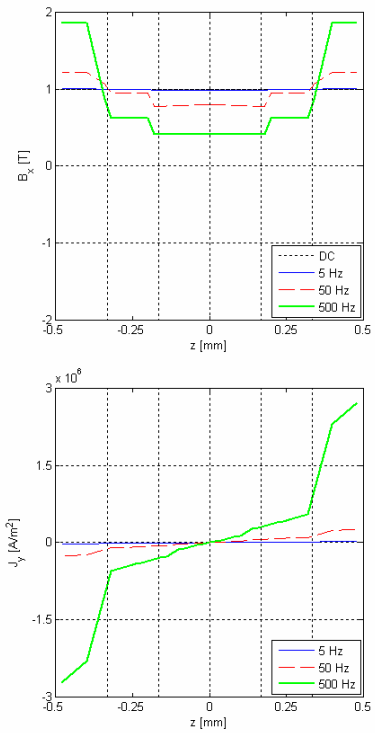

Fig. 4. The magnetic flux density and the eddy current density inside the specimen

The excitation generates an *x*-directed magnetic field and the resulting magnetic flux density and eddy current density can be seen in Fig. 4 along the line $x = 0$, $y = 0$, $z \in [-0.5,...,0.5]$mm. The penetration depth is 1.9mm, 0.6mm and 0.19mm in the case of *f*=5Hz, 50Hz, 500Hz.

4.2 Specimen with Magnetic Hysteresis

The first analysis is the rotational magnetic field without taking into account eddy currents. The specimen is magnetized in the *x* direction then the magnetic field is rotated by keeping the magnitude of current constant. The magnetic field intensity and the magnetic flux density vectors are plotted in Fig. 5. The effect of hysteresis characteristic can be seen in the figures, because there is a phase shift between the field quantity vectors.

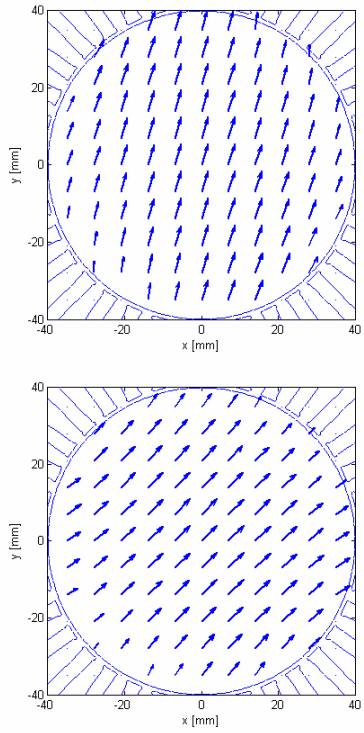

Fig. 5. The magnetic field intensity and the magnetic flux density vectors

The next analysis is to determine the variation of magnetic flux density in the case of linear excitation in the *x* direction. The magnetic flux density will be measured by *B*-coils which measure the averaged value of magnetic flux density and it is plotted in Fig. 6 for different values of frequency of excitation. The figure shows the situation when the magnetic flux density has a maximum value. The magnetic flux density is decreasing a little by increasing the frequency.

The magnetic flux density can be determined from the induced voltage measured by the *B*-coils. If the length of *B*-coils is 20mm and the number of turns is 5, then the peak value of the simulated induced voltage is 0.3mV and 30mV if the frequency is *f*=50Hz and *f*=500Hz, respectively.

The next analysis is the study of effect of eddy currents in rotational magnetic field. The specimen is magnetized in the *x* direction then the magnetic field is rotated by keeping the magnitude of current constant.

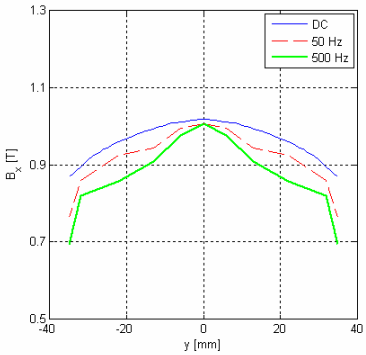

Fig. 6. Averaged flux density inside the specimen

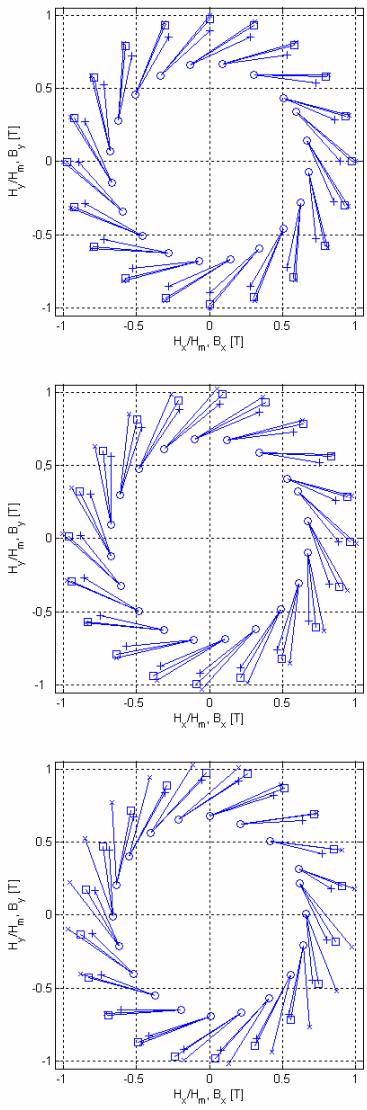

Fig. 7. The magnetic field intensity and the magnetic flux density vector in DC case, and when *f*=50Hz and *f*=500Hz

The magnetic field intensity and the magnetic flux density vector are plotted in Fig.7. The effect of hysteresis characteristic and the eddy currents can be seen in the figures, because there is an increasing phase between the field quantity vectors by increasing the frequency.

There are three values of magnetic flux density in the figures, they are corresponding to three *B*-coils with different length, +, □ and x sign the magnetic flux density determined by applying a coil with 70mm, 40mm and 20mm length, respectively. In these simulations the magnetic flux density has not been controlled.

5 CONCLUSIONS

The first results of FEM simulation of a RRSST system under construction are presented in this paper. This study generates some ongoing technical solutions and future works.

The most important revelation is that the implementation of an inverse vector hysteresis model is unescapable, because this results in a much faster solver. The application of direct vector hysteresis model in the inverse mode is very slow, because there are 4-5 steps in every direction to determine the appropriate value of the scalar magnetic field intensity, and there are 10 directions in the applied model, which is 40-50 steps in every tetrahedron placed in specimen, which could be saved.

It would be important to generate a much denser mesh which results in finer solutions. The application of prism elements especially inside the specimen would give better mesh to decrease the number of hysteresis models. The control of the waveform of magnetic flux density has not been implemented yet, because this modification would results in a time consuming procedure. That is why the FEM program must be made faster. Other type of potential formulation can also be implemented to select the best one.

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