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FRACTIONAL – ORDER FEEDBACK CONTROL OF A DC MOTOR

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This paper deals with the feedback control of a DC motor speed with using the fractional-order controller. The permanent-magnet DC motor is often used in mechatronic and other fields of control theory and therefore its control is very important. The mathematical description of the fractional - order controller and its implementation in the analogue and the discrete domains is presented. An example of simulation and possible realization of the particular case of digital fractional-order $PI^{\lambda}D^{\delta}$ controller are shown as well. The hardware realization is proposed in digital form with the microprocessor and in analogue form with the fractance circuits.

K e y w o r d s: fractional calculus, fractional-order controller, microprocessor, fractance, DC motor

1 INTRODUCTION

The DC motor is a power actuator, which converts direct current electrical energy into rotational mechanical energy. The DC motors are still often used in industry and in numerous control applications, robotic manipulators and commercial applications such as disk drive, tape motor as well.

We will consider the armature - controlled DC motor utilizes a constant field current. This kind of the DC motor will be controlled by a nonconventional control technique which is known as a fractional-order control. Mentioned technique was developed during last few decades and there are various practical applications as for example flexible spacecraft attitude control [25], car suspension control [29], temperature control [32], motor control [51], etc. This idea of the fractional calculus application to control theory was described in many other works (*eg*: [4], [15], [31], [38], *etc*) and its advantages were proved as well. All these works used the continuous models based on fractional differential equations or transfer function. For practical application of the fractional-order models in control and for realization of the fractional-order controllers (FOC), we need discrete fractional-order models. It is also well known that the fractional-order systems have an unlimited memory (infinite dimensional) while the integer-order systems have a limited memory (finite dimensional). It is important to approximately describe the fractional-order systems using a finite difference equations. We will consider new discretization technique proposed by Chen *et al* in [12]. Obtained discrete version of fractional order controller will be implemented by a microprocessor and proposed to the DC motor control.

This article is organized as follow: In section 2, we present a brief introduction to fractional calculus and its approximation. Section 3 presents mathematical model of DC motor as a controlled object. Section 4 deals with fractional order control. Section 5 presents some simulation results. Section 6 treats of proposal to digital and analogue realization of the FOC. Section 7 concludes this paper by some remarks and conclusions.

2 FUNDAMENTALS OF FRACTIONAL CALCULUS

2.1 A bit of history and definitions

Fractional calculus is a generalization of integration and differentiation to non-integer (fractional) order fundamental operator ${}_aD_t^r$, where a and t are the limits and $(r \in R)$ is the order of the operation. There are several definition of fractional integration and differentiation (see [28], [29], [39]). The most often used are the Grünwald-Letnikov (GL) definition and the Riemann-Liuville definition (RL). For a wide class of functions, the two definitions – GL and RL – are equivalent [39].

The GL is given as

$$ {}_aD_t^r f(t) = \lim_{h\to 0} h^{-r} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \binom{r}{j} f(t-jh), \qquad (1)$$

where $[\cdot]$ means the integer part. The RL definition is given as

$$ {}_aD_t^r f(t) = \frac{1}{\Gamma(n-r)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{r-n+1}} \mathrm{d}\tau \,, \qquad (2)$$

for $(n-1 < r < n)$ and where $\Gamma(\cdot)$ is the *Gamma* function.

For many engineering applications the Laplace transform methods are often used. The Laplace transform of the GL and RL fractional derivative/integral, under zero initial conditions for order r is given by [28]:

$$ \mathcal{L}\{ {}_aD_t^{\pm r} f(t); s\} = s^{\pm r} F(s)\,. \qquad (3)$$

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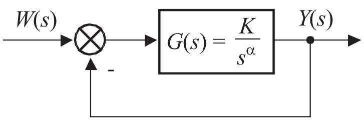

Fig. 1. Bode's ideal loop

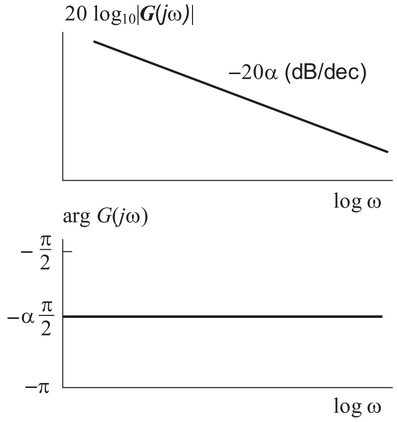

Fig. 2. Bode plots of transfer function $G_o(s)$ in (5)

Some other important properties of the fractional derivatives and integrals can be found in several works ([28],[29], [39], *etc*).

Geometric and physical interpretation of fractional integration and fractional differentiation were exactly described in [40].

2.2 Bode's ideal loop as a reference model

H. W. Bode suggested an ideal shape of the loop transfer function in his work on design of feedback amplifiers in 1945. Ideal loop transfer function has form [7]:

$$L(s) = \left(\frac{s}{\omega_{gc}}\right)^{\alpha}, \tag{4}$$

where ω_{gc} is desired crossover frequency and α is slope of the ideal cut-off characteristic.

Phase margin is $\Phi_m = \pi(1+\alpha/2)$ for all values of the gain. The amplitude margin A_m is infinity. The constant phase margin 60^o, 45^o and 30^o correspond to the slopes $\alpha = -1.33$, -1.5 and -1.66.

The Nyquist curve for ideal Bode transfer function is simply a straight line through the origin with $\arg(L(j\omega)) = \alpha\pi/2$.

Bode's transfer function (4) can be used as a reference system in the following form [3], [24], [36], [41], [50]:

$$G_c(s) = \frac{K}{s^{\alpha}+K}, \quad G_o(s) = \frac{K}{s^{\alpha}}, \quad (0 < \alpha < 2), \tag{5}$$

where $G_c(s)$ is transfer function of closed loop and $G_o(s)$ is transfer function in open loop.

General characteristics of Bode's ideal transfer function are:

(a) Open loop:

- Magnitude: constant slope of $-\alpha 20$ dB/dec;
- Crossover frequency: a function of K;
- Phase: horizontal line of $-\alpha\frac{\pi}{2}$;
- Nyquist: straight line at argument $-\alpha\frac{\pi}{2}$.

(b) Closed loop:

- Gain margin: $A_m = \infty$;
- Phase margin: constant : $\Phi_m = \pi\left(1-\frac{\alpha}{2}\right)$;
- Step response:

$$y(t) = Kt^{\alpha}E_{\alpha,\alpha+1}\left(-Kt^{\alpha}\right),$$

where $E_{a,b}(z)$ is the Mittag-Leffler function of two parameters [38].

2.3 Continuous time approximation of fractional calculus

A detailed review of the various approximation methods and techniques for continuous and discrete fractional-order models in form of IIR and FIR filters was done in work [45].

For simulation purpose, here we present the Oustaloup's approximation algorithm [29], [30]. The method is based on the approximation of a function of the form:

$$H(s) = s^r, \qquad r \in R, \qquad r \in [-1; 1] \tag{6}$$

for the frequency range selected as (ω_b, ω_h) by a rational function:

$$\widehat{H}(s) = C_o \prod_{k=-N}^{N} \frac{s+\omega'_k}{s+\omega_k} \tag{7}$$

using the following set of synthesis formulas for zeros, poles and the gain:

$$\omega'_k = \omega_b\left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+0.5(1-r)}{2N+1}},$$

$$\omega_k = \omega_b\left(\frac{\omega_h}{\omega_b}\right)^{\frac{k+N+0.5(1-r)}{2N+1}}, \tag{8}$$

$$C_o = \left(\frac{\omega_h}{\omega_b}\right)^{-\frac{r}{2}} \prod_{k=-N}^{N}\frac{\omega_k}{\omega'_k}, \tag{9}$$

where ω_h, ω_b are the high and low transitional frequencies. An implementation of this algorithm in Matlab as a function script `ora_foc()` is given in [14].

Using the described Oustaloup-Recursive-Approximation (ORA) method with:

$$\omega_h = 10^3, \quad \omega_b = 10^{-3}, \tag{10}$$

the obtained approximation for fractional function $H(s) = s^{-\frac{1}{2}}$ is:

$$\widehat{H_5}(s) = \frac{s^5+74.97s^4+768.5s^3+1218s^2+298.5s+10}{10s^5+298.5s^4+1218s^3+768.5s^2+74.97s+1}. \tag{11}$$

The Bode plots and the unit step response of the approximated fractional order integrator (11) are depicted in Fig. 3. Bode plots can be compared with the ideal plots depicted in Fig. 2.

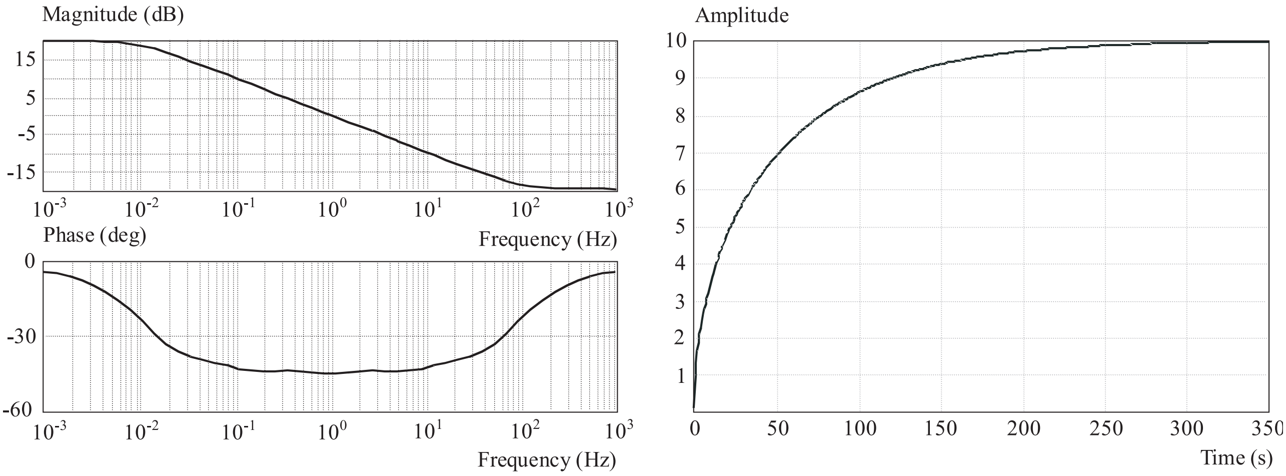

Fig. 3. Characteristics of approximated fractional order integrator (11): Bode plots for $r = -0.5$ and $N = 5$ (left), Unit step response for $r = -0.5$ and $N = 5$ (right)

2.4 Discrete time approximation of fractional calculus

In general, the discretization of fractional-order differentiator/integrator $s^{\pm r}$ $(r \in R)$ can be expressed by the so-called *generating function* $s \approx \omega(z^{-1})$. This generating function and its expansion determine both the form of the approximation and the coefficients [19].

As a generating function $\omega(z^{-1})$ can be used in generally the following formula [6]:

$$\omega(z^{-1}) = \Big(\frac{1}{\beta T}\frac{1-z^{-1}}{\gamma+(1-\gamma)z^{-1}}\Big), \tag{12}$$

where β and γ are denoted the gain and phase tuning parameters, respectively. For example, when $\beta = 1$ and $\gamma = \{0,\ 1/2,\ 7/8,\ 1,\ 3/2\}$, the generating function (12) becomes the forward Euler, the Tustin, the Al-Alaoui, the backward Euler, the implicit Adams rules, respectively. In this sense the generating formula can be tuned precisely.

The expansion of the generating functions can be done by Power Series Expansion (PSE) or Continued Fraction Expansion (CFE).

It is very important to note that PSE scheme leads to approximations in the form of polynomials, that is, the discretized fractional order derivative is in the form of FIR filters, which have only zeros.

Taking into account that our aim is to obtain discrete equivalents to the fractional integrodifferential operators in the Laplace domain, $s^{\pm r}$, the following considerations have to be made [46]:

1. s^r, $(0 < r < 1)$, viewed as an operator, has a branch cut along the negative real axis for arguments of s on $(-\pi, \pi)$ but is free of poles and zeros.
2. It is well known that, for interpolation or evaluation purposes, rational functions are sometimes superior to polynomials, roughly speaking, because of their ability to model functions with zeros and poles. In other words, for evaluation purposes, rational approximations frequently converge much more rapidly than PSE and have a wider domain of convergence in the complex plane.

In this paper, for directly discretizing s^r, $(0 < r < 1)$, we shall concentrate on the IIR form of discretization where as a generating function we will adopt an Al-Alaoui idea on mixed scheme of Euler and Tustin operators [1], [2] but we will use a different ration between both operators. The mentioned new operator, raised to power $\pm r$, has the form [34]:

$$(\omega(z^{-1}))^{\pm r} = \Big(\frac{1+a}{T}\frac{1-z^{-1}}{1+az^{-1}}\Big)^{\pm r}, \tag{13}$$

where a is ratio term and r is fractional order. The ratio term a is the amount of phase shift and this tuning knob is sufficient for most solved engineering problems.

In expanding the above in rational functions, we will use the CFE. It should be pointed out that, for control applications, the obtained approximate discrete-time rational transfer function should be stable and minimum phase. Furthermore, for a better fit to the continuous frequency response, it would be of high interest to obtain discrete approximations with poles an zeros interlaced along the line $z \in (-1, 1)$ of the z plane. The direct discretization approximations proposed in this paper enjoy the desirable properties.

The result of such approximation for an irrational function, $\widehat{G}(z^{-1})$, can be expressed by $G(z^{-1})$ in the CFE form [46]:

$$G(z^{-1}) \simeq \\ a_0(z^{-1}) + \cfrac{b_1(z^{-1})}{a_1(z^{-1}) + \cfrac{b_2(z^{-1})}{a_2(z^{-1}) + \cfrac{b_3(z^{-1})}{a_3(z^{-1}) + \dots}}} \\ = a_0(z^{-1}) + \frac{b_1(z^{-1})}{a_1(z^{-1})+}\ \frac{b_2(z^{-1})}{a_2(z^{-1})+}\ \dots \frac{b_3(z^{-1})}{a_3(z^{-1})+} \dots \tag{14}$$

where a_i and b_i are either rational functions of the variable z^{-1} or constants. The application of the method

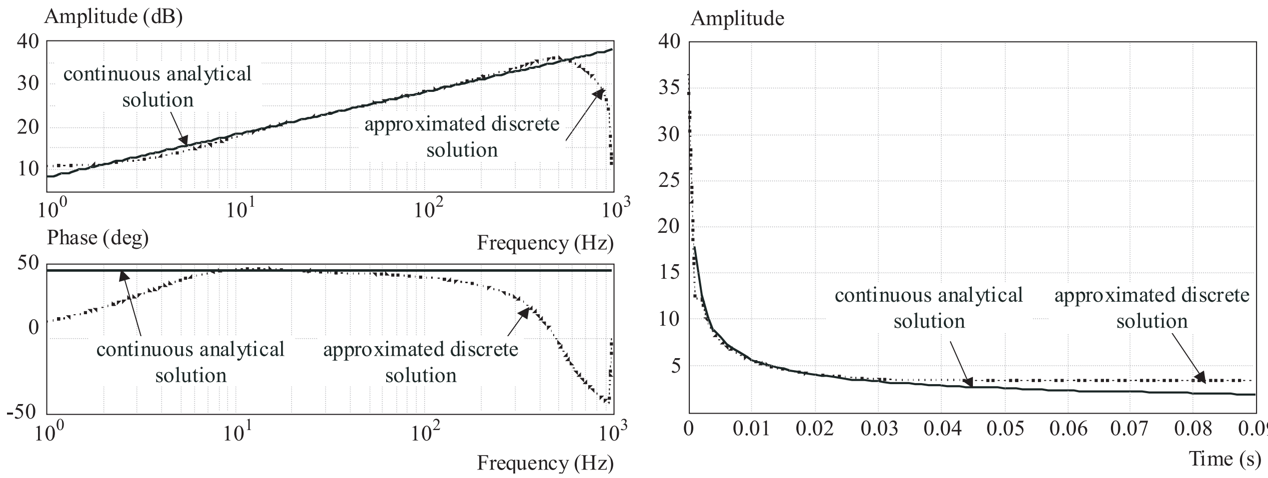

Fig. 4. Characteristics of approximated fractional order differentiator (16): Bode plots for $r = 0.5$, $n = 5$, $a = 1/3$, and $T = 0.001$ s in (15) (left), Unit step responses for $r = 0.5$, $n = 5$, $a = 1/3$, and $T = 0.001$ s in (15) (right)

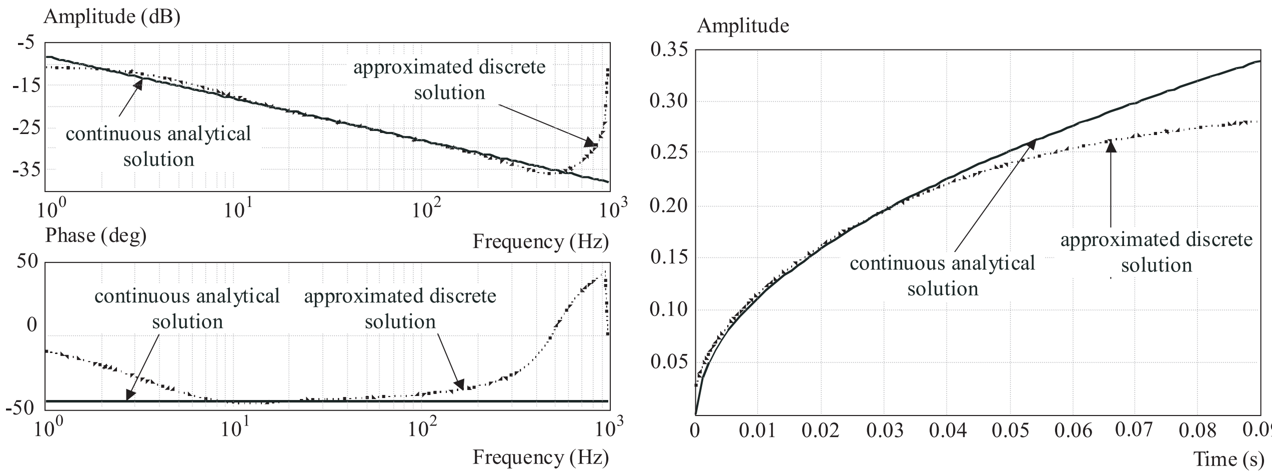

Fig. 5. Characteristics of approximated fractional order integrator (17): Bode plots for $r = -0.5$, $n = 5$, $a = 1/3$, and $T = 0.001$ s in (15) (left), Unit step responses for $r = -0.5$, $n = 5$, $a = 1/3$, and $T = 0.001$ s in (15) (right)

yields a rational function, $G(z^{-1})$, which is an approximation of the irrational function $\widehat{G}(z^{-1})$.

The resulting discrete transfer function, approximating fractional-order operators, can be expressed as:

$$
\begin{aligned}
(\omega(z^{-1}))^{\pm r} &\approx \Big(\frac{1+a}{T}\Big)^{\pm r} \mathrm{CFE}\Big\{\Big(\frac{1-z^{-1}}{1+az^{-1}}\Big)^{\pm r}\Big\}_{p,q} \\
&= \Big(\frac{1+a}{T}\Big)^{\pm r} \frac{P_p(z^{-1})}{Q_q(z^{-1})}, \\
&= \Big(\frac{1+a}{T}\Big)^{\pm r} \frac{p_0 + p_1 z^{-1} + \cdots + p_m z^{-p}}{q_0 + q_1 z^{-1} + \cdots + q_n z^{-q}},
\end{aligned}
\tag{15}
$$

where $\mathrm{CFE}\{u\}$ denotes the continued fraction expansion of u; p and q are the orders of the approximation and P and Q are polynomials of degrees p and q. Normally, we can set $p = q = n$.

In Matlab Symbolic Toolbox, by the following script, for a given n we can easily get the approximated direct discretization of fractional order derivative (let us denote that $x = z^{-1}$):

```
syms r a x;maple('with(numtheory)');
f = ((1-x)/(1+a*x))^r;;
n=5; n2=2*n;
maple(['cfe := cfrac(' char(f) ',x,n2);'])
pq=maple('P_over_Q := nthconver','cfe',n2)
p0=maple('P := nthnumer','cfe',n2)
q0=maple('Q := nthdenom','cfe',n2)
p=(p0(5:length(p0)));q=(q0(5:length(q0)));
p1=collect(sym(p),x)
q1=collect(sym(q),x)
```

Modified and improved digital fractional-order differentiator using fractional sample delay and digital integrator using recursive Romberg integration rule and fractional order delay as well has been described in [42].

Some others solutions for design IIR approximation using least-squares *eg*: the Padé approximation, the Prony's method and the Shranks' method were described in [6]. The Prony and Shranks methods can produce better approximations the widely used CFE method. The Padé and the CFE methods yield the same approximation (causal,

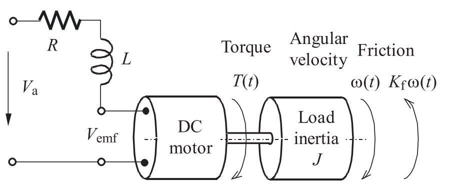

Fig. 6. General model of a DC motor

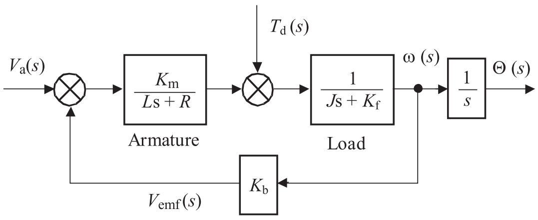

Fig. 7. Mathematical model of a DC motor

stable and minimum phase). Different approach of the CFE method was used in [23].

Here we present some results for fractional order $r = \pm 0.5$ (half order derivative/integral). The value of approximation order n is truncated to $n = 5$ and weighting factor a was chosen $a = 1/3$. Assume sampling period $T = 0.001$ s.

For $r = 0.5$ we have the following approximation of the fractional half-order derivative:

$$G(z^{-1}) = \frac{985.9 - 1315z^{-1} + 328.6z^{-2} + 36.51z^{-3}}{27 - 18z^{-1} - 3z^{-2} + z^{-3}} \qquad (16)$$

The Bode plots and unit step response of the digital fractional order differentiator (16) and the analytical continuous solution of a fractional semi-derivative are depicted in Fig. 4. Poles and zeros of the transfer function (16) lie in a unit circle.

For $r = -0.5$ we have the following approximation of the fractional half-order integral:

$$G(z^{-1}) = \frac{0.739 - 0.493z^{-1} - 0.0822z^{-2} + 0.0274z^{-3}}{27 - 36z^{-1} + 9z^{-2} + z^{-3}} \qquad (17)$$

The Bode plots and unit step response of the digital fractional order integrator (17) and the analytical continuous solution of a fractional semi-derivative are depicted in Fig. 5. Poles and zeros of the transfer function (17) lie in a unit circle.

3 MODEL OF A DC MOTOR

We will consider the general model of the DC motor (DCM) which is depicted in Fig. 6. The applied voltage V_a controls the angular velocity $\omega(t)$.

The relations for the armature controlled DC motor are shown schematically in Fig. 7. Transfer function (with $T_d(s) = 0$) has the form [16]:

$$G_{DCM}(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[(Ls+R)(Js+K_f) + K_bK_m]}. \qquad (18)$$

However, for many DCM the time constant of the armature is negligible and therefore we can simplify model (18). A simplified continuous mathematical model has the following form:

$$G_{DCM}(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[R(Js+K_f) + K_bK_m]} = \frac{[K_m/(RK_f + K_bK_m)]}{s(\tau s+1)} = \frac{K_{DCM}}{s(\tau s+1)}, \qquad (19)$$

where the time constant $\tau = RJ/(RK_f + K_bK_m)$ and $K_{DCM} = K_m/(RK_f + K_bK_m)$. It is of interest to note that $K_m = K_b$.

This mini DC motor with model number PPN13KA12C is great for robots, remote control applications, CD and DVD mechanics, etc. Specifications are [21]: min. voltage 1.5 V, nominal voltage 2 V, max. voltage 2.5 V, max. rated current 0.08 A, no load speed 3830 r/min and rated load speed 3315 r/min. For our mini DC motor the physical constants are: $R = 6\,\Omega$, $K_m = K_b = 0.1$, $K_f = 0.2\,\mathrm{N\,m\,s}$ and $J = 0.01\,\mathrm{kg\,m^2/s^2}$. For these motor constants the transfer function (19) of the DC motor has the form:

$$G_{DCM}(s) = \frac{0.08}{s(0.05s+1)}. \qquad (20)$$

Discrete mathematical model of the DC motor (20) obtained via new discretization method (13), for sampling period $T = 0.001$ s and $a = 1/3$, has the following form:

$$G_{DCM}(z^{-1}) = \frac{8.89 \times 10^{-3}z^{-2} + 0.053z^{-1} + 0.08}{8.844 \times 10^4 z^{-2} - 1.787 \times 10^5 z^{-1} + 9.022 \times 10^4}. \qquad (21)$$

In Fig. 8 is depicted the comparison of the continues (20) and dicsrete (21) model of the DC motor. As we can observe in figures, both models have a good agreement.

4 FRACTIONAL–ORDER CONTROL

4.1 Preliminary consideration

As we mentioned in introduction, we can also find works dealing with the application of the fractional calculus tool in control theory, but these works have usually theoretical character, whereas the number of works, in which a real object is analyzed and the fractional - order controller is designed and implemented in practice, is very small. The main reason for this fact is the difficulty of controller implementation. This difficulty arises from the mathematical nature of fractional operators, which, defined by convolution and implying a non-limited memory, demand hard requirements of processors memory and velocity capacities.

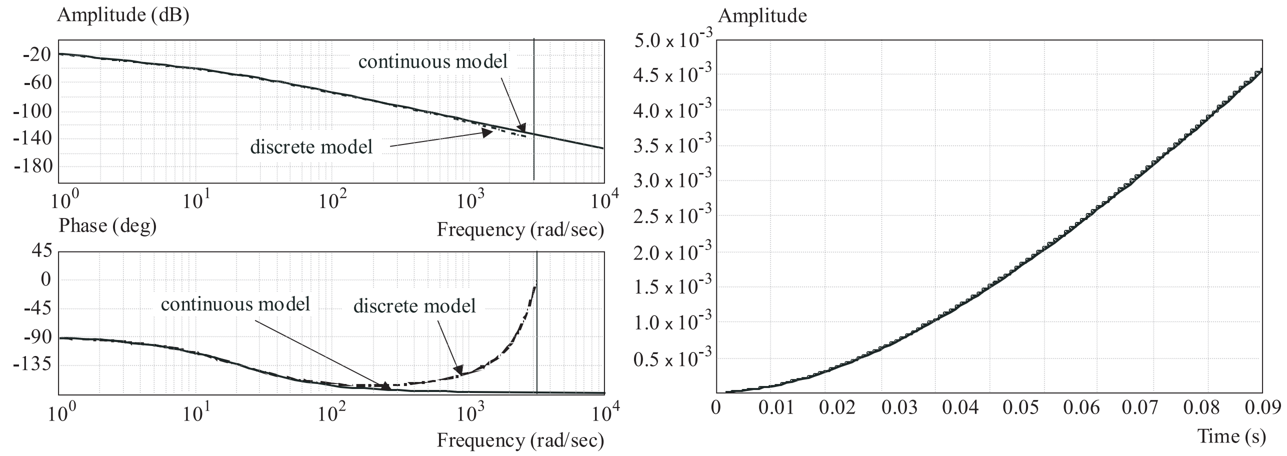

Fig. 8. Comparison of characteristics for both models of a DC motor: Bode plots of the motor models (left), Comparison of characteristics for both models of a DC motor (right)

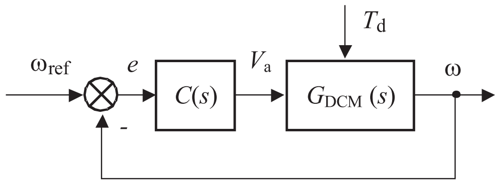

Fig. 9. Feedback control loop

4.2 Fractional-order controllers

The fractional-order $PI^\lambda D^\delta$ controller was proposed as a generalization of the PID controller with integrator of real order λ and differentiator of real order δ. The transfer function of such type the controller in Laplace domain has form [38]:

$$C(s) = \frac{U(s)}{E(s)} = K_p + K_i\, s^{-\lambda} + K_d\, s^{\delta}, \quad (\lambda, \delta > 0)\,, \quad (22)$$

where K_p is the proportional constant, K_i is the integration constant and K_d is the differentiation constant.

Transfer function (22) corresponds in discrete domain with the discrete transfer function in the following expression [46]:

$$C(z^{-1}) = \frac{U(z^{-1})}{E(z^{-1})} = K_p + K_i(\omega(z^{-1}))^{-\lambda} + K_d(\omega(z^{-1}))^{\delta}, \quad (23)$$

where λ and δ are arbitrary real numbers.

Taking $\lambda = 1$ and $\delta = 1$, we obtain a classical PID controller. If $\delta = 0$ and $K_d = 0$, we obtain a PI^λ controller, *etc.* All these types of controllers are particular cases of the $PI^\lambda D^\delta$ controller, which is more flexible and gives an opportunity to better adjust the dynamical properties of the fractional-order control system.

There are many another considerations of the fractional-order controller. For example we can notice the $CRONE$ controller [29], the non-integer integral and its application to control [24] or the TID compensator [20], which has a similar structure as a PID controller but the proportional component is replaced with a tilted component having a transfer function s to the power of $(-1/n)$.

All those fractional-order controllers are sometimes called optimal phase controllers because only with non-integer order we can get a constant phase somewhere between 0^o and -180^o depending on the parameters λ and δ.

4.3 Fractional-order controller design

For the FOC design we will use an idea which was proposed by Bode [7] and for first time used to the motion control described by Tustin [43]. This principle was also used by Manabe to induction motor speed control [25].

The several methods and tuning techniques for the FOC parameters were developed during the past ten years. They are based on various approaches (see [5], [13], [22], [26], [31], [49], [52]).

In Fig. 9 is depicted feedback control loop, where C(s) is transfer function of controller and $G_{DCM}(s)$ is transfer function of the DC motor.

We will design the controller, which give us a step response of feedback control loop with overshoot independent of payload changes (iso-damping). In the frequency domain point of view it means phase margin independent of the payload changes.

Phase margin of controlled system is [9], [48]

$$\Phi_m = \arg\left[C(j\omega_g)G_{DCM}(j\omega_g)\right] + \pi\,, \quad (24)$$

where $j\omega_g$ is the crossover frequency. Independent phase margin means in other words constant phase. This can be accomplished by controller of the form

$$C(s) = k_1 \frac{k_2 s + 1}{s^{\mu}}\,, \quad k_1 = 1/K_{DCM}\,, \quad k_2 = \tau\,. \quad (25)$$

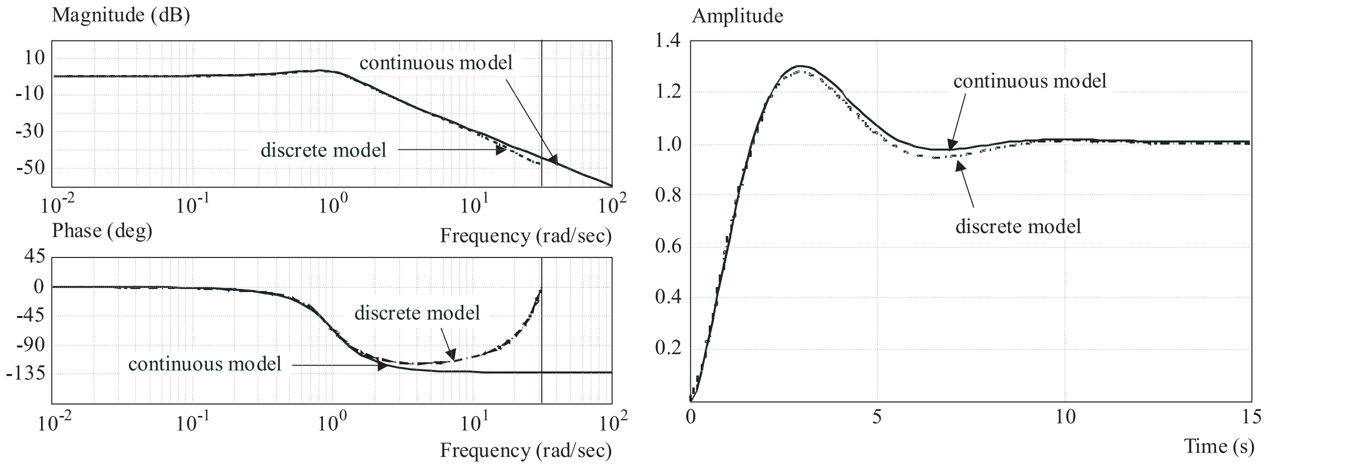

Fig. 10. Characteristics of fractional order transfer function (28): Bode plots – continuous and discrete models $n=5$, $a=1/3$, and $T=0.1$ s (left), Unit step responses –continuous and discrete models $n=5$, $a=1/3$, and $T=0.1$ s (right)

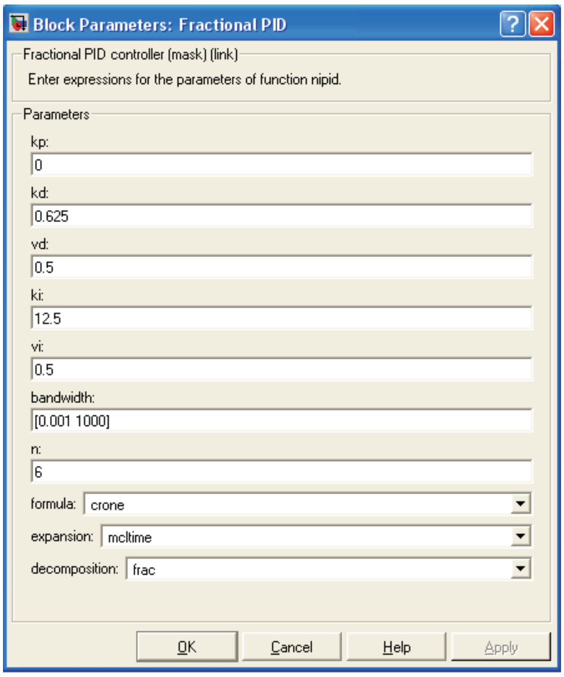

Fig. 11. Simulink block **nipid** - fractional order controller

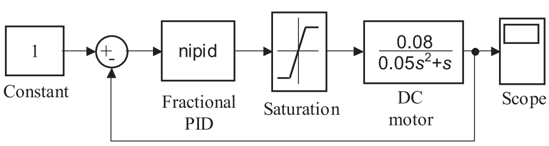

Fig. 12. Simulink model for feedback control of the DC motor

Such controller gives a constant phase margin and obtained phase margin is

$$\begin{aligned}\Phi_m &= \arg\left[C(j\omega)G_{DCM}(j\omega)\right]+\pi\\ &= \arg\left[\frac{k_1K_{DCM}}{(j\omega)^{(1+\mu)}}\right]+\pi\\ &= \arg\left[(j\omega)^{-(1+\mu)}\right]+\pi=\pi-(1+\mu)\frac{\pi}{2}\,. \end{aligned}\tag{26}$$

For our parameters of controlled object (20) and desired phase margin $\Phi_m = 45^o$, we get the following constants of the fractional order controllres (25): $k_1 = 12.5$, $k_2 = 0.05$ and $\mu = 0.5$. With these constants we obtain a fractional $I^\lambda D^\delta$ controller, which is a particular case of the $PI^\lambda D^\delta$ controller and has the form

$$\begin{aligned}C(s) &= \frac{\tau}{K_{DCM}}s^{0.5}+\frac{1}{K_{DCM}s^{0.5}}\\ &= K_ds^{0.5}+K_is^{-0.5}=0.625\sqrt{s}+\frac{12.5}{\sqrt{s}}\,,\end{aligned}\tag{27}$$

where $K_i = 12.5$, $K_d = 0.625$ and $\delta = \lambda = 0.5$.

According to relation (26), by using a controller (27), we can obtain a phase margin

$$\Phi_m = \arg\left[C(j\omega)G_{DCM}(j\omega)\right]+\pi=\pi-(1.5)\,\frac{\pi}{2}=45^\circ,$$

which was desired phase margin specification.

5 SIMULATION RESULTS

The transfer function of the closed feedback control loop with the fractional-order controller (27) and the DC motor (20) has the following form:

$$\begin{aligned}G_c(s) = \frac{G_o(s)}{1+G_o(s)} &= \frac{G_{DCM}(s)C(s)}{1+G_{DCM}(s)C(s)}\\ &= \frac{0.05s+1}{0.05s^{2.5}+s^{1.5}+0.05s+1}\,,\end{aligned}\tag{28}$$

where $G_o(s)$ is the transfer function of the open control loop with

$$G_o(s)=\frac{0.05s+1}{0.05s^{2.5}+s^{1.5}}\,.$$

The feedback control loop described above can be simulated in Matlab environment with using the approximation technique described before, namely Oustaloup's re-

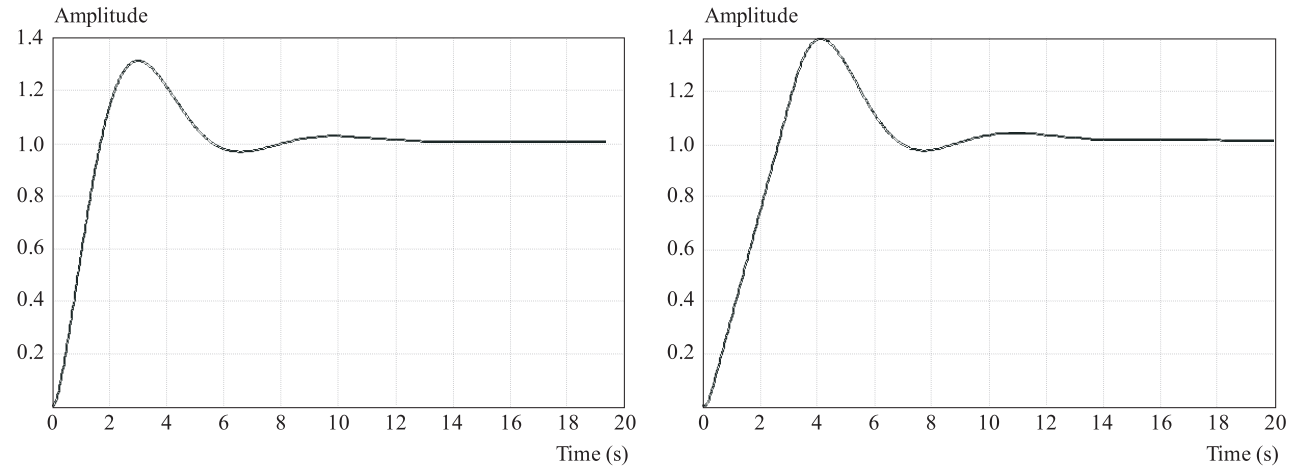

Fig. 13. Comparison of unit step responses of a feedback control loop: Unit step response without actuator saturation overshoot $\approx 30\,\%$, set. time ≈ 11 s (left), Unit step response with actuator saturation overshoot $\approx$ 40 %, settling time $\approx$ 14 s (right)

cursive approximation function `ora_foc()` for the desired frequency range given in (10).

```
close all; clear all;
Gs_DCM=tf([0.08],[0.05 1 0]);
Cs=(0.625*ora_foc(0.5,6,0.001,1000))
       +(12.5*ora_foc(-0.5,6,0.001,1000));
Gs_close=(Gs_DCM*Cs)/(1+(Gs_DCM*Cs));
step(Gs_close,15);
Gs_open=(Gs_DCM*Cs);
bode(Gs_open);
[Gm,Pm] = margin(Gs_open);
```

The results obtained via described Matlab scripts are depicted in Fig. 10. Continues model is shown with solid line. Phase margin is $\Phi_m \approx 44.9^\circ$ and gain margin is infinite.

The disrete version of the continues fractional order transfer function can be obtained with using the digital operator (13) and Matlab function for approximation of digital fractional order derivative/integral `dfod1()`. Assume that $T = 0.1$ s and $a = 1/3$.

```
close all; clear all;
T=0.1;
a=1/3;
z=tf('z',T,'variable','z^-1')
Hz=((1+a)/T)*((1-z^-1)/(1+a*z^-1));
Gz_DCM=0.08/(Hz*(0.05*Hz+1));
Cz=0.625*dfod1(5,T,a,0.5)+12.5*dfod1(5,T,a,-0.5);
Gz_close=(Gz_DCM*Cz)/(1+(Gz_DCM*Cz));
step(Gz_close,15);
Gz_open=(Gz_DCM*Cz);
bode(Gz_open);
[Gm,Pm] = margin(Gz_open);
```

The results obtained via described Matlab scripts are depicted in Fig. 10. Discrete model is shown with dashed line. Phase margin is $\Phi_m \approx 45.1^\circ$ and gain margin is infinite.

Simulation of the closed feedback loop can also be dome in Matlab/Simulink environment, where fractional - order controller is realized via `nipid` block proposed by D. Valerio [44], where block parameters are depicted in Fig. 11.

General Simulink model is shown in Fig. 12. Block constants were set according to parameters of DC motor and fractional-order controller.

Time domain simulation results for fractional order feedback loop are depicted in Fig. 13. Obtained results are comparable with the results obtained with simulation in Matlab by routines.

Stability analysis is investigated by solving the characteristic equation of transfer function (28) with using Matlab function `solve()`

```
s=solve('0.05*s^2.5 + s^1.5 + 0.05*s + 1 = 0','s')
```

with the following results: $s_{1,2} = -0.5 \pm 0.86602j$ and $s_3 = -20$. It means that feedback control loop is stable.

As we can observe in Fig. 13, the quality indexes (overshoot and settling time) are worse in the case of control loop with saturation, because of controller power limitations.

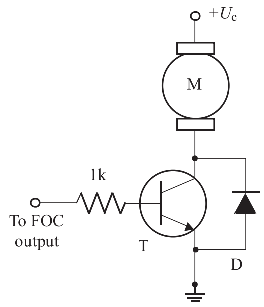

Fig. 14. Actuator for the DC motor

6 PROPOSED REALIZATIONS OF FOC

Basically, there are two methods for realization of the FOC. One is a digital realization based on processor devices and appropriate control algorithm and the second

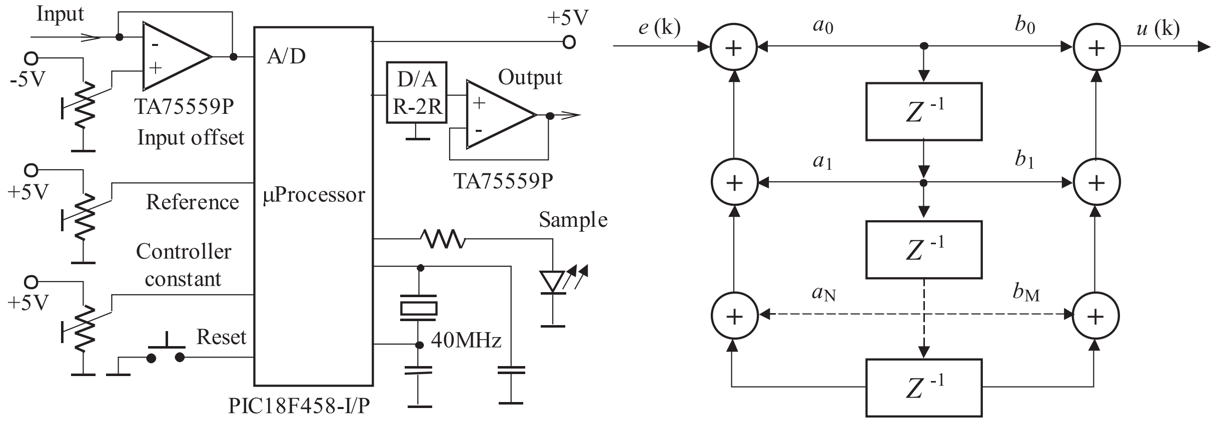

Fig. 15. Proposal for digital implementation of the FOC: Block diagram of the digital fractional-order controller based on PIC processor (left), Block diagram of the canonical representation of IIR filter form (right)

one is an analogue realization based on analogue circuits so-called *fractance*. In this section is described both of them.

In Fig. 14 is depicted the actuator for connection the DC motor to the FOC.

6.1 Digital realization: Control algorithm and HW

This realization can be based on implementation of the control algorithm in the processor devices, e.g.: PLC controller [35], processor C51 or PIC [33], PCL IO card [47], etc. Suppose that processor PIC18F458 is used [55]. Some experimental measurements with this processor were already done in [33].

Generally, the control algorithm is be based on canonical form of IIR filter, which can be expressed as follow

$$F(z^{-1}) = \frac{U(z^{-1})}{E(z^{-1})} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}\,, \qquad (29)$$

where $a_0 = 1$ for compatible with the definitions used in Matlab. Normally, we choose $M = N$.

For designed fractional-order controller (27) we can use the half-order approximations (16) and (17), respectively. The resulting discrete transfer function of the fractional-order controller arranged to canonical form (29) is represented as

$$C(z^{-1}) = \big(23.17 - 61.33z^{-1} + 55.87z^{-2} - 18.52z^{-3} + 0.268z^{-4} + 0.560z^{-5} + 0.032z^{-6}\big)\Big/\big(1.00 - 2.00z^{-1} + 1.11z^{-2} - 0.111z^{-4} + 0.0082z^{-5} + 0.0014z^{-6}\big) \qquad (30)$$

This controller can be directly implemented to any processor based devices as for instance PLC or PIC depicted in Fig. 15 left. A direct form of such implementation using canonical form shown in Fig. 15 right with input $e(k)$ and output $u(k)$ range mapping to the interval $0 - U_{FOC}$ [V] is divided into two sections: initialization code and cyclic code. Pseudocode has the following syntax

```
(* initialization code *)
scale := 32752; % input and output
order := 6; % order of approximation
U_FOC := 5; % input and output voltage range: 5[V], 10[V],
...
a[0] := 1.0; a[1] := -2.0; a[2] := 1.11; a[3] := 0.0;
a[4] := -0.111; a[5] := 0.0082; a[6] := 0.0014;
b[0] := 23.17; b[1] := -61.33; b[2] := 55.87; b[3] := -18.52;
b[4] := 0.268; b[5] := 0.560; b[6] := 0.032;

loop i := 0 to order do
s[i] := 0;
endloop

(* cyclic code *)
in := (REAL(input)/scale) * U_FOC;
feedback := 0; feedforward := 0;
loop i:=1 to order do
feedback := feedback - a[i] * s[i];
feedforward := feedforward + b[i] * s[i];
endloop
s[0] := in + a[0] * feedback;
out := b[0] * s[1] + feedforward;
loop i := order downto 1 do
s[i] := s[i-1];
endloop output := INT(out*scale)/U_FOC;
```

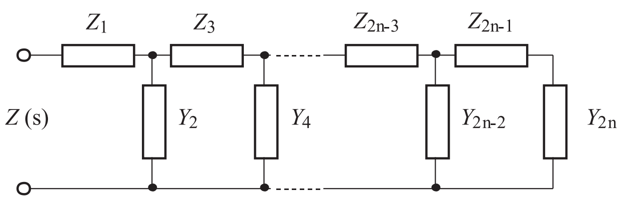

Fig. 16. Finite ladder circuit

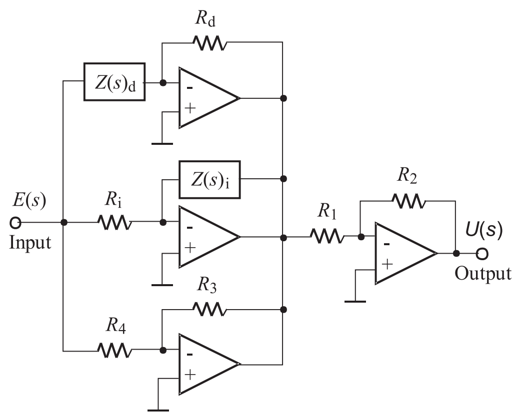

Fig. 17. Analogue fractional-order $PI^{\lambda}D^{\delta}$ controller

The disadvantage with this solution is that the complete controller is calculated using floating point arithmetic.

There are many softwares for PIC programming. As for example: Microchip MPLAB, HiTech C Compiler, PICBasic Pro, *etc.*

6.2 Analogue realization: Fractance circuits and fractor

A circuit exhibiting fractional-order behavior is called a *fractance* [39]. The fractance devices have the following characteristics [27], [28], [18]. First the phase angle is constant independent of the frequency within a wide frequency band. Second it is possible to construct a filter which has moderated characteristics which can not be realized by using the conventional devices.

Generally speaking, there are three basic fractance devices. The most popular is a domino ladder circuit network. Very often used is a tree structure of electrical elements and finally, we can find out also some transmission line circuit. Here we must mention that all basic electrical elements (resistor, capacitor and coil) are not ideal [10], [54].

Design of fractances can be done easily using any of the rational approximations [36] or a truncated CFE, which also gives a rational approximation.

Truncated CFE does not require any further transformation; a rational approximation based on any other methods must be transformed to the form of a continued fraction. The values of the electric elements, which are necessary for building a fractance, are then determined from the obtained finite continued fraction. If all coefficients of the obtained finite continued fraction are positive, then the fractance can be made of classical passive elements (resistors and capacitors). If some of the coefficients are negative, then the fractance can be made with the help of negative impedance converters [37].

Domino ladder lattice networks can approximate fractional operator more effectively than the lumped networks [17].

Let us consider the circuit depicted in Fig. 16, where $Z_{2k-1}(s)$ and $Y_{2k}(s)$, $k = 1, \dots, n$, are given impedances of the circuit elements. The resulting impedance $Z(s)$ of the entire circuit can be found easily, if we consider it in the right-to-left direction:

$$Z(s) = Z_1(s) + \cfrac{1}{Y_2(s) + \cfrac{1}{Z_3(s) + \cfrac{1}{Y_4(s) + \cfrac{\cdots\cdots\cdots}{\cfrac{1}{Y_{2n-2}(s) + \cfrac{1}{Z_{2n-1}(s) + \cfrac{1}{Y_{2n}(s)}}}}}}} \tag{31}$$

The relationship between the finite domino ladder network, shown in Fig. 16, and the continued fraction (31) provides an easy method for designing a circuit with a given impedance $Z(s)$. For this one has to obtain a continued fraction expansion for $Z(s)$. Then the obtained particular expressions for $Z_{2k-1}(s)$ and $Y_{2k}(s)$, $k = 1, \dots, n$, will give the types of necessary components of the circuit and their nominal values.

Rational approximation of the fractional integrator/differentiator can be formally expressed as

$$s^{\pm\alpha} \approx \left\{ \frac{P_p(s)}{Q_q(s)} \right\}_{p,q} = Z(s)\,, \tag{32}$$

where p and q are the orders of the rational approximation, P and Q are polynomials of degree p and q, respectively.

For direct calculation of circuit elements was proposed method by Wang [53]. This method was designed for constructing resistive-capacitive ladder network and transmission lines that have a generalized Warburg impedance $As^{-\alpha}$, where A is independent of the angular frequency and $0 < \alpha < 1$. This impedance may appear at an electrode/electrolyte interface, etc. The impedance of the ladder network (or transmission line) can be evaluated and rewritten as a continued fraction expansion:

$$Z(s) = R_0 + \frac{1}{C_0 s +} \frac{1}{R_1 +} \frac{1}{C_1 s +} \frac{1}{R_2 +} \frac{1}{C_2 s +} \cdots \tag{33}$$

If we consider that $Z_{2k-1} \equiv R_{k-1}$ and $Y_{2k} \equiv C_{k-1}$ for $k = 1, \dots, n$ in Fig. 16, then the values of the resistors and capacitors of the network are specified by

$$\begin{aligned} R_k &= 2h^{\alpha} P(\alpha) \frac{\Gamma(k+\alpha)}{\Gamma(k+1-\alpha)} - h^{\alpha}\delta_{ko} \\ C_k &= h^{1-\alpha}(2k+1) \frac{\Gamma(k+1-\alpha)}{P(\alpha)\Gamma(k+1+\alpha)}\,, \\ P(\alpha) &= \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)}\,, \end{aligned} \tag{34}$$

where $0 < \alpha < 1$, h is an arbitrary small number, δ_{ko} is the Kronecker delta, and k is an integer, $k \in [0, \infty)$.

In Fig. 17 is depicted an analogue implementation of fractional-order $PI^\lambda D^\delta$ controller. Fractional order differentiator is approximated by general Warburg impedance $Z(s)_d$ and fractional order integrator is approximated by impedance $Z(s)_i$, where orders of both approximations are $0 < \alpha < 1$. For orders greater than 1, the Warburg impedance can be combined with classical integer order one. Usually we suppose $R_2 = R_1$ in Fig. 17. For proportional gain K_p we can write the formula

$$K_p = \frac{R_3}{R_4}.$$

The integration and derivation constants K_i and K_d can be computed from relationships

$$K_i = \frac{Z(s)_i}{R_i}, \qquad K_d = \frac{R_d}{Z(s)_d}.$$

In the case, if we use identical resistors (R-series) and identical capacitors (C-shunt) in the fractances, then the behavior of the circuit will be as a half-order integrator/differentiator. Realization and measurements of such kind controllers were done in [36]. Some others experimental results we can find in [11].

Instead fractance circuit the new electrical element introduced by G. Bohannan which is so-called *fractor* can be used as well [8]. This element — fractor made from a material with the properties of $LiN_2H_5SO_4$ has been already used for temperature control [5].

7 CONCLUSION

In this paper was presented a case study of fractional order feedback control of a DC motor. Described method is based on Bode's ideal control loop. Design algorithm for fractional-order $PI^\lambda D^\delta$ controller parameters uses a phase margin specification of open control loop. Another very important advantage is an iso-damping property of such control loop. Simulation results obtained via Matlab/Simulink confirm the described theoretical suggestion. This article also proposed digital and analogue realization of fractional-order controller. Described techniques are useful for practical implementation of fractional-order controllers as the non-conventional control techniques. However this approach also gave a good start for analysis and design of the analog fractional order controller. The fractional-order controller gives us an insight into the concept of memory of the fractional order operator. The design, realization, and implementation of the fractional order control systems also became possible and much easier than before.

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