Riešenie 2D Laplaceovej rovnice separáciou premenných v cylindrickej sústave, pri $\frac{\partial^2\varphi}{\partial z^2}=0$

$$\Delta\varphi(\rho,\psi)=\frac{\partial}{\rho\,\partial\rho}\left(\rho\frac{\partial\varphi}{\partial\rho}\right)+\frac{\partial^2\varphi}{\rho^2\,\partial\psi^2}=0$$

$$\varphi(\rho,\psi)=R(\rho)\Phi(\psi)$$

$$\Delta\varphi=\frac{\partial}{\rho\,\partial\rho}\left(\rho\frac{\partial R\Phi}{\partial\rho}\right)+\frac{\partial^2 R\Phi}{\rho^2\,\partial\psi^2}=0,\quad \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho R'\Phi\right)+\frac{R\Phi''}{\rho^2}=0$$

$$\frac{\Delta\varphi}{R\Phi}=\frac{\rho}{R}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho R'\right)+\frac{\Phi''}{\Phi}=0\Leftrightarrow\begin{cases}\frac{\rho}{R}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho R'\right)=k^2 \quad\Rightarrow\quad R(\rho)=a\rho^k+b\rho^{-k},\quad k\neq 0\\ \frac{\Phi''}{\Phi}=-k^2\quad\Rightarrow\quad \Phi(\psi)=c\cos(k\psi)+d\sin(k\psi),\quad k\neq 0\\ \\ \frac{\rho}{R}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho R'\right)=0\quad\Rightarrow\quad a_0+b_0\ln\rho\\ \frac{\Phi''}{\Phi}=0\quad\Rightarrow\quad \mathrm{c}_0\psi+d_0\end{cases}$$

$$\varphi_0(\rho,\psi)=A_0+B_0\ln\rho+C_0\psi+D_0\psi\ln\rho$$

$$\varphi_n(\rho,\psi)=R_n(\rho)\Phi_n(\psi),\quad n=1,2,3,\ldots$$

$$\varphi_n(\rho,\psi)=A_n\rho^n\cos(n\psi)+B_n\rho^{-n}\cos(n\psi)+C_n\rho^n\sin(n\psi)+D_n\rho^{-n}\sin(n\psi)$$

Všeobecné riešenie potom je:

$$\varphi(\rho,\psi)=\varphi_0(\rho,\psi)+\sum_{n=1}^{\infty}\varphi_n(\rho,\psi)=A_0+B_0\ln\rho+C_0\psi+D_0\psi\ln\rho+$$

$$+\sum_{n=1}^{\infty}\left[A_n\rho^n+B_n\rho^{-n}\right]\cos(n\psi)+\sum_{n=1}^{\infty}\left[C_n\rho^n+D_n\rho^{-n}\right]\sin(n\psi)$$

Po úprave tak, aby bola zaručená konvergencia radov, dostávame možné riešenia – pre dve rôzne (ale priľahlé) oblasti:

$$\varphi(\rho,\psi)=a_0+b_0\ln\frac{\rho}{\rho_0}+\sum_{n=1}^{\infty}(\frac{\rho}{\rho_0})^n\left[a_n\cos(n\psi)+b_n\sin(n\psi)\right],\quad \text{pri } \rho<\rho_0$$

$$\varphi(\rho,\psi)=a_0+b_0\ln\frac{\rho}{\rho_0}+\sum_{n=1}^{\infty}(\frac{\rho_0}{\rho})^n\left[a_n\cos(n\psi)+b_n\sin(n\psi)\right],\quad \text{pri } \rho>\rho_0$$

Neznáme koeficienty $a_0, a_\mathrm{n}, b_\mathrm{n}$ určíme napríklad na základe známeho priebehu potenciálu (Dirichlet) na hranici, t.j. pri $\rho=\rho_0$,

$$\varphi(\rho_0,\psi)=a_0+\sum_{n=1}^{\infty}\left[a_n\cos(n\psi)+b_n\sin(n\psi)\right]$$

takto:

$$a_0=\frac{1}{2\pi}\int_0^{2\pi}\varphi(\rho_0,\psi)d\psi,\quad a_\mathrm{n}=\frac{1}{2\pi}\int_0^{2\pi}\varphi(\rho_0,\psi)\cos(n\psi)d\psi,\quad b_\mathrm{n}=\frac{1}{2\pi}\int_0^{2\pi}\varphi(\rho_0,\psi)\sin(n\psi)d\psi$$