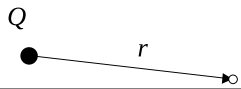
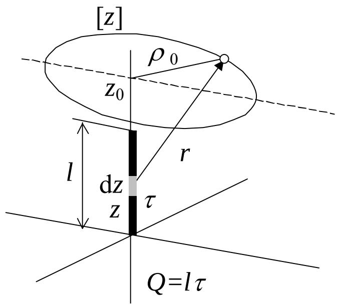


$$\varphi(r)=\frac{1}{4\pi\varepsilon}\frac{Q}{r}$$



$$\mathrm{d}\varphi(\rho_0,z_0)=\frac{\mathrm{d}Q}{4\pi\varepsilon r},\quad \mathrm{d}Q=\tau\mathrm{d}z,\quad r=\sqrt{\rho_0^2+(z_0-z)^2}$$

$$\varphi(\rho_0,z_0)=\frac{\tau}{4\pi\varepsilon}\int_0^l\frac{\mathrm{d}z}{\sqrt{\rho_0^2+(z_0-z)^2}}=\frac{\tau}{4\pi\varepsilon}\int_{z_0-l}^{z_0}\frac{\mathrm{d}\xi}{\sqrt{\rho_0^2+\xi^2}}=$$

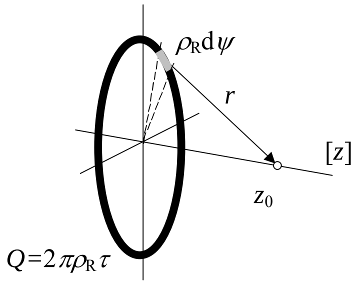
$$\frac{\tau}{4\pi\varepsilon}\ln\frac{z_0+\sqrt{\rho_0^2+z_0^{\,2}}}{z_0-l+\sqrt{\rho_0^2+(z_0-l)^2}}=\frac{Q}{4\pi\varepsilon l}\ln\frac{z_0+\sqrt{\rho_0^2+z_0^{\,2}}}{z_0-l+\sqrt{\rho_0^2+(z_0-l)^2}}$$

$$\varphi(\infty,\frac{l}{2})\to\frac{Q}{4\pi\varepsilon l}\ln\frac{1+\frac{l}{2\rho_0}}{1-\frac{l}{2\rho_0}}=\frac{Q}{4\pi\varepsilon l}\ln\frac{1+\frac{l}{2\rho_0}}{1-\frac{l}{2\rho_0}}\to\frac{Q}{4\pi\varepsilon l}2\left(\frac{l}{2\rho_0}\right)=\frac{Q}{4\pi\varepsilon\rho_0}$$

Špeciálny prípad, v mieste $z_0=l/2$ (v rovine symetrie) a ak $l\to\infty$ vedie k výsledku ktorý zodpovedá nekonečne dlhému líniovému zdroju:

$$\varphi(\rho)=\frac{\tau}{4\pi\varepsilon}\ln\frac{\frac{l}{2}+\sqrt{\rho^2+(\frac{l}{2})^2}}{-\frac{l}{2}+\sqrt{\rho^2+(\frac{l}{2})^2}}=\frac{\tau}{4\pi\varepsilon}\ln\frac{1+\sqrt{1+(\frac{2\rho}{l})^2}}{-1+\sqrt{1+(\frac{2\rho}{l})^2}}\doteq\frac{\tau}{4\pi\varepsilon}\ln\frac{1+\left(1+\frac{1}{2}(\frac{2\rho}{l})^2\right)}{-1+\left(1+\frac{1}{2}(\frac{2\rho}{l})^2\right)}=$$

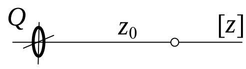
$$=\frac{\tau}{4\pi\varepsilon}\ln\frac{2+\frac{1}{2}(\frac{2\rho}{l})^2}{\frac{1}{2}(\frac{2\rho}{l})^2}\to\frac{\tau}{4\pi\varepsilon}\ln\frac{2}{\frac{1}{2}(\frac{2\rho}{l})^2}=\frac{\tau}{4\pi\varepsilon}\ln\frac{4l^2}{\rho^2}=\underbrace{\frac{\tau}{4\pi\varepsilon}\ln 4l^2}_{konst.}-\frac{\tau}{4\pi\varepsilon}\ln\rho^2=-\frac{\tau}{2\pi\varepsilon}\ln\rho$$

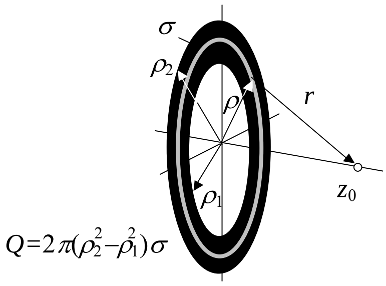


$$\mathrm{d}\varphi(z_0)=\frac{\mathrm{d}Q}{4\pi\varepsilon r},\quad \mathrm{d}Q=\tau\rho_R\mathrm{d}\psi,\quad r=\sqrt{\rho_R^2+z_0^2}$$

$$\varphi(z_0)=\frac{\tau\rho_R}{4\pi\varepsilon}\int_0^{2\pi}\frac{\mathrm{d}\psi}{\sqrt{\rho_R^2+z_0^2}}=\frac{\tau\rho_R}{2\varepsilon\sqrt{\rho_R^2+z_0^2}}=\frac{Q}{4\pi\varepsilon\sqrt{\rho_R^2+z_0^2}}$$

$$\varphi(\infty)\to\frac{Q}{4\pi\varepsilon z_0}$$





$$\mathrm{d}Q=\sigma\mathrm{d}s,\ \mathrm{d}s=2\pi\rho\mathrm{d}\rho,\quad r=\sqrt{\rho^2+z_0^2}$$

$$\mathrm{d}\varphi(z_0)=\frac{dQ}{4\pi\varepsilon\sqrt{\rho^2+z_0^2}}=\frac{\sigma}{2\varepsilon}\frac{\rho\mathrm{d}\rho}{\sqrt{\rho^2+z_0^2}}$$

$$\varphi(z_0)=\frac{\sigma}{2\varepsilon}\int_{\rho_1}^{\rho_2}\frac{\rho\mathrm{d}\rho}{\sqrt{\rho^2+z_0^2}}=\frac{\sigma}{2\varepsilon}\left[\sqrt{\rho^2+z_0^2}\right]_{\rho_1}^{\rho_2}$$

pri $\rho_1=0$ bude: $\varphi(z)=\frac{\sigma}{2\varepsilon}\left[\sqrt{\rho_2^2+z^2}-z\right]$

pri $\rho_2\to\infty$ bude: $\varphi(z)=\frac{\sigma}{2\varepsilon}\left[\rho_2-z\right]\ \to-\frac{\sigma z}{2\varepsilon}$,

lebo ρ_2 je konst.

Vypočítajte vo všetkých prípadoch aj intenzitu elektrostatického poľa, ako –gradient skalárneho potenciálu.