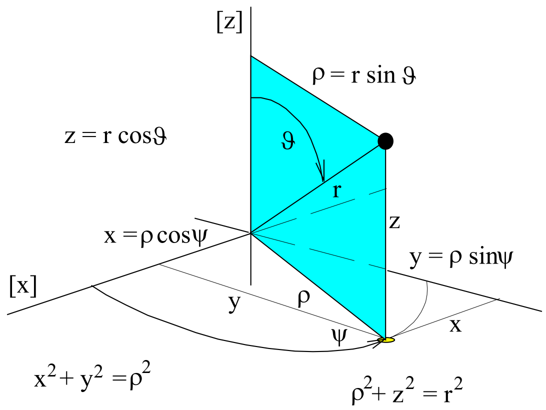


Metrické koeficienty - (Lamé)

Sústava	h_1	h_2	h_3	p_1, p_2, p_3
Karteziánska	1	1	1	x, y, z
Cylindrická	1	ρ	1	ρ, ψ, z
Sférická	1	r	ρ	r, ϑ, ψ

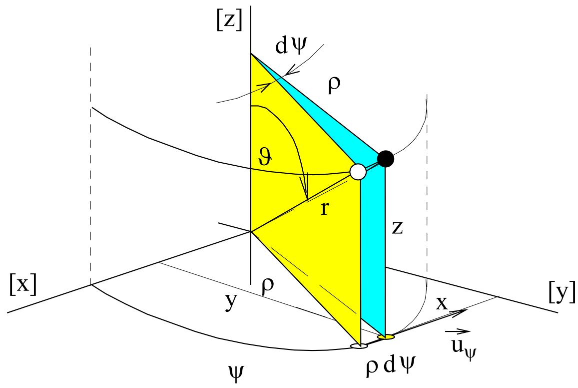
dĺžkové elementy: $dl_1 = h_1.p_1$, $dl_2 = h_2.p_2$, $dl_3 = h_3.p_3$

plošné elementy: $ds_1 = h_2.dp_2\ h_3.dp_3$

$ds_2 = h_3.dp_3\ h_1.dp_1$

$ds_3 = h_1.dp_1\ h_2.dp_2$

objemový element: $dv = h_1.dp_1\ h_2.dp_2\ h_3.dp_3$



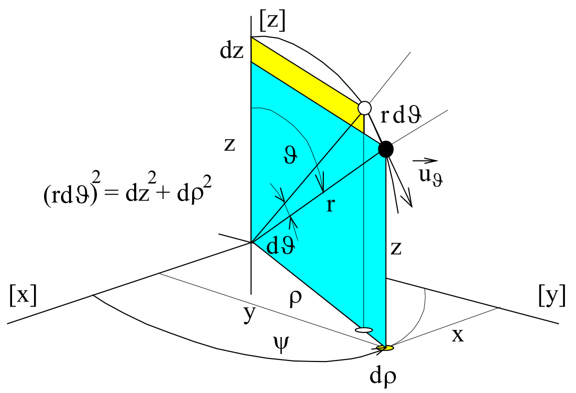
$$\varphi \equiv \varphi(p_1, p_2, p_3)$$

$$grad\varphi = \vec{\nabla}\varphi = \vec{\mathbf{u}}_1 \frac{\partial\varphi}{h_1\partial p_1} + \vec{\mathbf{u}}_2 \frac{\partial\varphi}{h_2\partial p_2} + \vec{\mathbf{u}}_3 \frac{\partial\varphi}{h_3\partial p_3}$$

Laplaceov operátor - aplikovaný na skalárnu funkciu

$$div(grad\varphi) = \vec{\nabla}\cdot\vec{\nabla}\varphi = \nabla^2\varphi = \Delta\varphi =$$

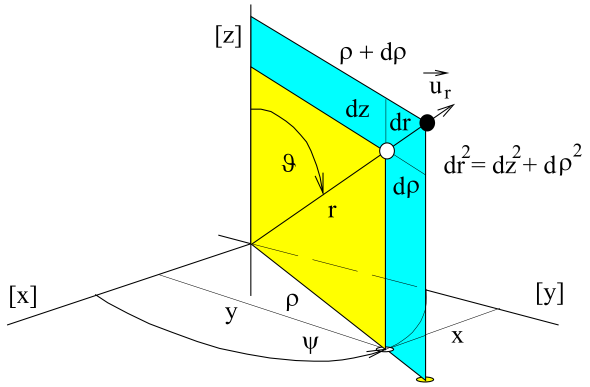
$$= \frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial p_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\varphi}{\partial p_1}\right) + \frac{\partial}{\partial p_2}\left(\frac{h_3h_1}{h_2}\frac{\partial\varphi}{\partial p_2}\right) + \frac{\partial}{\partial p_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\varphi}{\partial p_3}\right)\right)$$



$$\vec{\mathbf{F}} = \vec{\mathbf{u}}_1 F_1(p_1,p_2,p_3) + \vec{\mathbf{u}}_2 F_2(p_1,p_2,p_3) + \vec{\mathbf{u}}_3 F_3(p_1,p_2,p_3)$$

$$div\vec{\mathbf{F}} = \vec{\nabla}\cdot\vec{\mathbf{F}} = \frac{1}{h_1h_2h_3}\left(\frac{\partial(h_2h_3F_1)}{\partial p_1} + \frac{\partial(h_3h_1F_2)}{\partial p_2} + \frac{\partial(h_1h_2F_3)}{\partial p_3}\right)$$

$$rot\vec{\mathbf{F}} = \vec{\nabla}\times\vec{\mathbf{F}} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\vec{\mathbf{u}}_1 & h_2\vec{\mathbf{u}}_2 & h_3\vec{\mathbf{u}}_3 \\ \frac{\partial}{\partial p_1} & \frac{\partial}{\partial p_2} & \frac{\partial}{\partial p_3} \\ h_1F_1 & h_2F_2 & h_3F_3 \end{vmatrix}$$



Laplaceov operátor aplikovaný na vektorovú funkciu

$\vec{\mathbf{F}}(x,y,z)$ - vzorec platí len v karteziánskej sústave

$$\vec{\nabla}\cdot\vec{\nabla} = \nabla^2\vec{\mathbf{F}} = \vec{\mathbf{u}}_x\nabla^2F_x + \vec{\mathbf{u}}_y\nabla^2F_y + \vec{\mathbf{u}}_z\nabla^2F_z$$

$$div(rot\vec{\mathbf{F}}) \equiv 0$$

$$rot(grad\varphi) \equiv 0$$