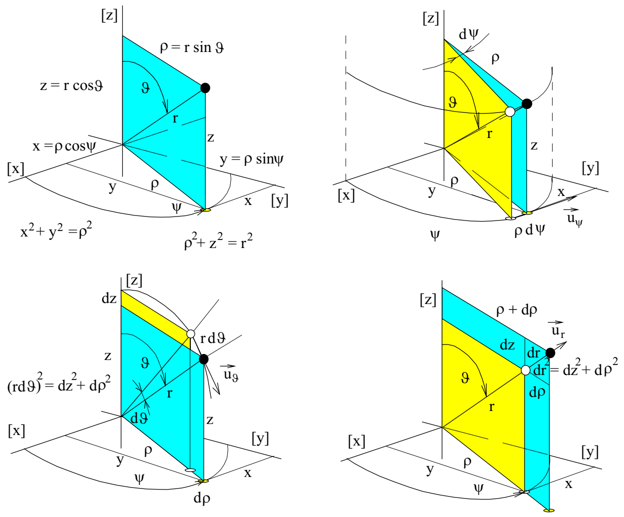

$$\varphi \equiv \varphi(p_1, p_2, p_3)$$

$$grad\varphi = \vec{\nabla}\varphi = \vec{\mathbf{u}}_1 \frac{\partial\varphi}{h_1\partial p_1} + \vec{\mathbf{u}}_2 \frac{\partial\varphi}{h_2\partial p_2} + \vec{\mathbf{u}}_3 \frac{\partial\varphi}{h_3\partial p_3}$$

Laplaceov operátor - aplikovaný na skalárnu funkciu

$$div(grad\varphi) = \vec{\nabla}\cdot\vec{\nabla}\varphi = \nabla^2\varphi = \Delta\varphi =$$

$$= \frac{1}{h_1h_2h_3}\left(\frac{\partial}{\partial p_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\varphi}{\partial p_1}\right) + \frac{\partial}{\partial p_2}\left(\frac{h_3h_1}{h_2}\frac{\partial\varphi}{\partial p_2}\right) + \frac{\partial}{\partial p_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\varphi}{\partial p_3}\right)\right)$$

$$\vec{\mathbf{F}} = \vec{\mathbf{u}}_1 F_1(p_1,p_2,p_3) + \vec{\mathbf{u}}_2 F_2(p_1,p_2,p_3) + \vec{\mathbf{u}}_3 F_3(p_1,p_2,p_3)$$

$$div\vec{\mathbf{F}} = \vec{\nabla}\cdot\vec{\mathbf{F}} = \frac{1}{h_1h_2h_3}\left(\frac{\partial(h_2h_3F_1)}{\partial p_1} + \frac{\partial(h_3h_1F_2)}{\partial p_2} + \frac{\partial(h_1h_2F_3)}{\partial p_3}\right)$$

$$rot\vec{\mathbf{F}} = \vec{\nabla}\times\vec{\mathbf{F}} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\vec{\mathbf{u}}_1 & h_2\vec{\mathbf{u}}_2 & h_3\vec{\mathbf{u}}_3 \\ \frac{\partial}{\partial p_1} & \frac{\partial}{\partial p_2} & \frac{\partial}{\partial p_3} \\ h_1F_1 & h_2F_2 & h_3F_3 \end{vmatrix}$$

Laplaceov operátor aplikovaný na vektorovú funkciu

$\vec{\mathbf{F}}(x,y,z)$ - vzorec platí len v karteziánskej sústave

$$\vec{\nabla}\cdot\vec{\nabla} = \nabla^2\vec{\mathbf{F}} = \vec{\mathbf{u}}_x\nabla^2F_x + \vec{\mathbf{u}}_y\nabla^2F_y + \vec{\mathbf{u}}_z\nabla^2F_z$$

$$div(rot\vec{\mathbf{F}}) \equiv 0$$

$$rot(grad\varphi) \equiv 0$$

Metrické koeficienty

Sústava	h_1	h_2	h_3	p_1, p_2, p_3
Karteziánska	1	1	1	x, y, z
Cylindrická	1	ρ	1	ρ, ψ, z
Sférická	1	r	ρ	r, ϑ, ψ

dĺžkové elementy: $dl_1 = h_1.p_1$, $dl_2 = h_2.p_2$, $dl_3 = h_3.p_3$

plošné elementy: $ds_1 = h_2.dp_2\ h_3.dp_3$

$ds_2 = h_3.dp_3\ h_1.dp_1$

$ds_3 = h_1.dp_1\ h_2.dp_2$

objemový element: $dv = h_1.dp_1\ h_2.dp_2\ h_3.dp_3$

Partikulárne riešnia dvojrozmernej Laplaceovej rovnice - metóda separácie premenných

Karterziánska súradnicová sústava	Cylindrická súradnicová sústava	Sférická súradnicová sústava
$\varphi(x,y) = X(x)Y(y)$	$\varphi(\rho,\psi) = R(\rho)\Psi(\psi)$	$\varphi(r,\vartheta) = R(r)\Theta(\vartheta)$
$X(x) = Ae^{kx} + Be^{-kx}$	$R(\rho) = A\rho^n + B\rho^{-n}$	$R(r) = Ar^n + Br^{-(n+1)}$
$Y(y) = C\cos(ky) + D\sin(ky)$	$\Psi(\psi) = C\cos(n\psi) + D\sin(n\psi)$	$\Theta(\vartheta) = CP_n(\cos\vartheta)$
$k \neq 0$	$n = 1,2,3,4\ldots$	$n = 0,1,2,3,\ldots$
pri $k = 0$	pri n = 0	$P_0 = 1$
$X(x) = ax + b$	$R(\rho) = a\ln\rho + b$	$P_1 = \cos\vartheta$
$Y(y) = cy + d$	$\Psi(\psi) = c\psi + d$	$P_2 = 1.5\cos^2\vartheta - 0.5$
		⋮

Riešnie vlnovej rovnice - homogénna rovinná vlna v karteziánskej súradnicovej sústave:

$$\vec{\mathcal{E}}_d(\vec{\mathbf{r}}) = \vec{\mathcal{E}}_{d0}e^{-\gamma_1\vec{\mathbf{n}}_d\cdot\vec{\mathbf{r}}},\quad \vec{\mathcal{H}}_d(\vec{\mathbf{r}}) = \vec{\mathcal{H}}_{d0}e^{-\gamma_1\vec{\mathbf{n}}_d\cdot\vec{\mathbf{r}}},\quad \vec{\mathcal{E}}_{d0} = \vec{\mathbf{n}}_d\times\vec{\mathcal{H}}_{d0}Z_1,\quad \vec{\mathcal{H}}_{d0} = \vec{\mathbf{n}}_d\times\vec{\mathcal{E}}_{d0}/Z_1$$

$$\vec{\mathcal{E}}_r(\vec{\mathbf{r}}) = \vec{\mathcal{E}}_{r0}e^{-\gamma_1\vec{\mathbf{n}}_r\cdot\vec{\mathbf{r}}},\quad \vec{\mathcal{H}}_r(\vec{\mathbf{r}}) = \vec{\mathcal{H}}_{r0}e^{-\gamma_1\vec{\mathbf{n}}_r\cdot\vec{\mathbf{r}}},\quad \vec{\mathcal{E}}_{r0} = \vec{\mathbf{n}}_r\times\vec{\mathcal{H}}_{r0}Z_1,\quad \vec{\mathcal{H}}_{r0} = \vec{\mathbf{n}}_r\times\vec{\mathcal{E}}_{r0}/Z_1$$

$$\vec{\mathcal{E}}_p(\vec{\mathbf{r}}) = \vec{\mathcal{E}}_{p0}e^{-\gamma_2\vec{\mathbf{n}}_p\cdot\vec{\mathbf{r}}},\quad \vec{\mathcal{H}}_p(\vec{\mathbf{r}}) = \vec{\mathcal{H}}_{p0}e^{-\gamma_2\vec{\mathbf{n}}_p\cdot\vec{\mathbf{r}}},\quad \vec{\mathcal{E}}_{p0} = \vec{\mathbf{n}}_p\times\vec{\mathcal{H}}_{p0}Z_2,\quad \vec{\mathcal{H}}_{p0} = \vec{\mathbf{n}}_p\times\vec{\mathcal{E}}_{p0}/Z_2$$

- kolmý dopad vlny na rovinné rozhranie:

$$\vec{\mathcal{E}}(x) = \vec{\mathcal{E}}_d(x) + \vec{\mathcal{E}}_r(x) = \vec{\mathcal{E}}_{d0}e^{-\gamma x} + \vec{\mathcal{E}}_{r0}e^{\gamma x} \qquad \mathcal{E}(x) = \mathcal{E}_d(x) + \mathcal{E}_r(x) = \mathcal{E}_{d0}e^{-\gamma x} + \mathcal{E}_{r0}e^{\gamma x}$$

$$\vec{\mathcal{H}}(x) = \vec{\mathcal{H}}_d(x) + \vec{\mathcal{H}}_r(x) = \vec{\mathcal{H}}_{d0}e^{-\gamma x} + \vec{\mathcal{H}}_{r0}e^{\gamma x} \qquad \mathcal{H}(x) = \mathcal{H}_d(x) \pm \mathcal{H}_r(x) = \mathcal{H}_{d0}e^{-\gamma x} \pm \mathcal{H}_{r0}e^{\gamma x}$$

$$\rho_E(x) = \frac{\mathcal{E}_r(x)}{\mathcal{E}_d(x)} \qquad \rho_H(x) = \frac{\mathcal{H}_r(x)}{\mathcal{H}_d(x)} \qquad \frac{\mathcal{E}(x)}{\mathcal{H}(x)} = \frac{\mathcal{E}_d(x) \mp \mathcal{E}_r(x)}{\mathcal{H}_d(x) \pm \mathcal{H}_r(x)} = \frac{\mathcal{E}_d(x)[1 \mp \rho_E(x)]}{\mathcal{H}_d(x)[1 \pm \rho_H(x)]} \qquad Z_0 = \frac{\mathcal{E}_d(x)}{\mathcal{H}_d(x)} = \frac{\mathcal{E}_r(x)}{\mathcal{H}_r(x)}$$

- šikmý dopad vlnenia na rovinné rozhranie: Snellov zákon - $\gamma_1\sin\vartheta_1 = \gamma_2\sin\vartheta_2$

$$\begin{Bmatrix} \mathcal{E} = \mathcal{E}_d + \mathcal{E}_r \\ \mathcal{H}_t = \mathcal{H}_{td} - \mathcal{H}_{tr} \end{Bmatrix} \quad \tau_\perp = \frac{\mathcal{E}_{p0}}{\mathcal{E}_{d0}} = \frac{2Z_2\cos\vartheta_1}{Z_2\cos\vartheta_1 + Z_1\cos\vartheta_2}, \quad \rho_\perp = \frac{\mathcal{E}_{r0}}{\mathcal{E}_{d0}} = \frac{Z_2\cos\vartheta_1 - Z_1\cos\vartheta_2}{Z_2\cos\vartheta_1 + Z_1\cos\vartheta_2}, \quad \tau_\perp - \rho_\perp = 1$$

$$\begin{Bmatrix} \mathcal{H} = \mathcal{H}_d + \mathcal{H}_r \\ \mathcal{E}_t = \mathcal{E}_{td} - \mathcal{E}_{tr} \end{Bmatrix} \quad \tau_\parallel = \frac{\mathcal{H}_{p0}}{\mathcal{H}_{d0}} = \frac{2Z_1\cos\vartheta_1}{Z_1\cos\vartheta_1 + Z_2\cos\vartheta_2}, \quad \rho_\parallel = \frac{\mathcal{H}_{r0}}{\mathcal{H}_{d0}} = \frac{Z_1\cos\vartheta_1 - Z_2\cos\vartheta_2}{Z_1\cos\vartheta_1 + Z_2\cos\vartheta_2}, \quad \tau_\parallel - \rho_\parallel = 1$$